

# Identity Logic

$r=l$  = Romeo is the lover of Juliet. (identity)

$Ir$  = Romeo is Italian. (predication)

$(\exists x)Ix$  = There are Italians. (existence)

The result of writing a small letter and then “=” and then a small letter is a wff.

Romeo isn't the lover of Juliet =  $\sim r=l$

Someone besides Romeo is Italian =  $(\exists x)(\sim x=r \cdot Ix)$   
Someone who isn't Romeo is Italian

Romeo alone is Italian =  $(Ir \cdot \sim(\exists x)(\sim x=r \cdot Ix))$   
Romeo is Italian but no one else is

There is exactly one Italian =  $(\exists x)(Ix \cdot \sim(\exists y)(\sim y=x \cdot Iy))$

There are at least two Italians =  $(\exists x)(\exists y)((Ix \cdot Iy) \cdot \sim x=y)$

There are exactly two Italians =  $(\exists x)(\exists y)((Ix \cdot Iy) \cdot \sim x=y) \cdot \sim(\exists z)((\sim z=x \cdot \sim z=y) \cdot Iz)$

$$1 + 1 = 2$$

If exactly one being is F  
and exactly one being is G  
and nothing is F-and-G,  
then exactly two beings  
are F-or-G.

$$\begin{aligned} & (((\exists x)(Fx \cdot \sim(\exists y)(\sim y=x \cdot Fy)) \\ & \cdot (\exists x)(Gx \cdot \sim(\exists y)(\sim y=x \cdot Gy))) \\ & \cdot \sim(\exists x)(Fx \cdot Gx)) \supset \\ & (\exists x)(\exists y)((\exists z)((Fx \vee Gx) \cdot (Fy \vee Gy)) \cdot (\sim x=y \\ & \cdot \sim(\exists z)((\sim z=x \cdot \sim z=y) \cdot (Fz \vee Gz)))) \end{aligned}$$

# Identity Principles

Self-identity  
axiom

$$a=a$$

Substitute-equals  
rule

$$a=b, Fa \rightarrow Fb$$

There's more than one being. (pluralism)  
 $\therefore$  It's false that there's exactly one being. (monism)

- \* 1  $(\exists x)(\exists y)\sim x=y$  Valid
- [  $\therefore \sim(\exists x)(y)y=x$
- \* 2 asm:  $(\exists x)(y)y=x$
- \* 3  $\therefore(\exists y)\sim a=y$  {from 1}
- 4  $\therefore\sim a=b$  {from 3}
- 5  $\therefore(y)y=c$  {from 2}
- 6  $\therefore a=c$  {from 5}
- 7  $\therefore b=c$  {from 5}
- 8  $\therefore a=b$  {from 6 and 7}
- 9  $\therefore \sim(\exists x)(y)y=x$  {from 2; 4 contradicts 8}

# Do we need to qualify the substitute-equals rule?

Jones believes that Lincoln is on the penny.

Lincoln is the first Republican president.

∴ Jones believes that the first Republican president is on the penny.

B1

l=r

∴ Br

# Relational Logic

$Lrj$  = Romeo loves Juliet.

$Bxyz$  = x is between y and z.

The result of writing a capital letter and then two or more small letters is a wff.

Juliet loves Romeo =  $Ljr$   
Juliet loves herself =  $Ljj$   
Juliet loves Romeo but not Paris =  $(Ljr \cdot \sim Ljp)$

Everyone loves him/herself =  $(x)Lxx$   
Someone loves him/herself =  $(\exists x)Lxx$   
No one loves him/herself =  $\sim(\exists x)Lxx$

Someone (everyone,  
no one) loves Romeo

=

For some (all, no) x,  
x loves Romeo.

Normally put  
quantifiers  
*before* relations.

Romeo loves someone  
(everyone, no one)

=

For some (all, no) x,  
Romeo loves x.

Someone loves Romeo =  $(\exists x)Lxr$

For some x, x loves Romeo

Everyone loves Romeo =  $(x)Lxr$

For all x, x loves Romeo

No one loves Romeo =  $\sim(\exists x)Lxr$

It's not the case that, for  
some x, x loves Romeo

Romeo loves someone =  $(\exists x)Lrx$

For some x, Romeo loves x

Romeo loves everyone =  $(x)Lrx$

For all x, Romeo loves x

Romeo loves no one =  $\sim(\exists x)Lrx$

It's not the case that, for  
some x, Romeo loves x

Some Montague loves Juliet =  $(\exists x)(Mx \cdot Lxj)$

For some x,	x is a Montague and	x loves Juliet
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All Montagues love Juliet =  $(x)(Mx \supset Lxj)$

For all x,	if x is a Montague then	x loves Juliet
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Romeo loves some Capulet =  $(\exists x)(Cx \cdot Lrx)$

For some x,	x is a Capulet and	Romeo loves x
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Romeo loves all Capulets =  $(x)(Cx \supset Lrx)$

For all x,	if x is a Capulet then	Romeo loves x
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Some Montague besides Romeo loves Juliet =  $(\exists x)((Mx \cdot \sim x=r) \cdot Lxj)$

For some x,	x is a Montague and x isn't Romeo and	x loves Juliet
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Romeo loves all Capulets besides Juliet =  $(x)((Cx \cdot \sim x=j) \supset Lrx)$

For all x,	if x is a Capulet and x isn't Juliet then	Romeo loves x
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Romeo loves all Capulets who love themselves =  $(x)((Cx \cdot Lxx) \supset Lrx)$

For all x,	if x is a Capulet and x loves x then	Romeo loves x
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These have two different relations:

All who know Juliet love Juliet =  $(x)(Kxj \supset Lxj)$

For all x,	if x knows Juliet then	x loves Juliet
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All who know themselves love themselves =  $(x)(Kxx \supset Lxx)$

For all x,	if x knows x then	x loves x
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# Translate these now.

1. God loves Ignatius.
2. Everyone loves God.
3. God loves everyone.
4. All Jesuits love God.
5. God loves some Jesuits.
6. God loves everyone who doesn't love himself.
7. God loves all Jesuits who don't love themselves.
8. All Jesuits love themselves.
9. Ignatius loves everyone besides himself.
10. Some Jesuits love some besides themselves.

## These have two quantifiers:

Someone loves someone =  $(\exists x)(\exists y)Lxy$

For some x and for some y,	x loves y
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Everyone loves everyone =  $(x)(y)Lxy$

For all x and for all y,	x loves y
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Every Montague hates every Capulet =  $(x)(y)((Mx \cdot Cy) \supset Hxy)$

For all x and for all y,	if x is a Montague and y is a Capulet then	x hates y
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Everyone loves someone.

For all x there's some y,  
such that x loves y.

$$(\forall x)(\exists y)Lxy$$

There's someone who everyone loves.

There's some y such that,  
for all x, x loves y.

$$(\exists y)(\forall x)Lxy$$

Every Capulet loves some Montague =  $(x)(Cx \supset (\exists y)(My \cdot Lxy))$

For all x,	if x is a Capulet then	x loves some Montague
		for some y, y is a Montague and x loves y

Every Capulet loves someone =  $(x)(Cx \supset (\exists y)Lxy)$

For all x,	if x is a Capulet then	x loves someone
		for some y, x loves y

Everyone loves some Montague =  $(x)(\exists y)(My \cdot Lxy)$

For all x,	x loves some Montague
	for some y, y is a Montague and x loves y

Some Capulet loves every Montague =  $(\exists x)(Cx \cdot (y)(My \supset Lxy))$

For some x,	x is a Capulet and	x loves every Montague
		for all y, if y is a Montague then x loves y

Some Capulet loves everyone =  $(\exists x)(Cx \cdot (y)Lxy)$

For some x,	x is a Capulet and	x loves everyone
		for all y, x loves y

Someone loves every Montague =  $(\exists x)(y)(My \supset Lxy)$

For some x,	x loves every Montague
	for all y, if y is a Montague then x loves y

There is an unloved lover =  $(\exists x)(\sim(\exists y)Lyx \cdot (\exists y)Lxy)$

For some x,	x is unloved	and	x is a lover
	it is false that for some y, y loves x		for some y, x loves y

Everyone loves every lover =  $(x)((\exists y)Lxy \supset (y)Lyx)$

For all x,	if x is a lover	then	everyone loves x
	if, for some y, x loves y		for all y, y loves x

Romeo loves all and only those who don't love themselves

$$= (x)(Lrx \equiv \sim Lxx)$$

For all x,	Romeo loves x	if and only if	x doesn't love x
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All who know any person love that person =  $(x)(y)(Kxy \supset Lxy)$

For all x and for all y,	if x knows y then	x loves y
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## Reflexive / Irreflexive

Everyone loves himself =  $(x)Lxx$

No one loves himself =  $(x)\sim Lxx$

## Symmetrical / Asymmetrical

Universally, if x loves y then =  $(x)(y)(Lxy \supset Lyx)$

y loves x [does not love x] =  $(x)(y)(Lxy \supset \sim Lyx)$

## Transitive / Intransitive

Universally, if x loves y and y loves =  $(x)(y)(z)((Lxy \cdot Lyz) \supset Lxz)$

z, then x loves z [does not love z] =  $(x)(y)(z)((Lxy \cdot Lyz) \supset \sim Lxz)$

# Translate these now.

1. Some Jesuits love everyone.
2. No one loves all Franciscans.
3. All Jesuits love someone.
4. There is someone that all Jesuits love.
5. There is some Franciscan that everyone loves.
6. Some Franciscans love all Jesuits.
7. No Jesuits love all Franciscans.
8. Ignatius loves all and only those who don't love themselves.

## Translate these now.

1. Every Capulet loves some Montague.
2. Universally, if  $x$  knows  $y$  then  $x$  loves  $y$ .
3. There is an unloved lover.
4. Everyone loves all lovers.
5. Some Jesuits besides Ignatius love God.

1. Every Capulet loves some Montague.

$$(\forall x)(Cx \supset (\exists y)(My \cdot Lxy))$$

2. Universally, if x knows y then x loves y.

$$(\forall x)(\forall y)(Kxy \supset Lxy)$$

3. There is an unloved lover.

$$(\exists x)(\sim(\exists y)Lyx \cdot (\exists y)Lxy)$$

4. Everyone loves all lovers.

$$(\forall x)((\exists y)Lxy \supset (\forall y)Lyx)$$

5. Some Jesuits besides Ignatius love God.

$$(\exists x)((Jx \cdot \sim x=i) \cdot Lxg)$$

Hans loves Olga =  $Lho$

Hans loves someone =  $(\exists x)Lhx$

Hans loves some Russian =  $(\exists x)(Rx \cdot Lhx)$

Someone loves some Russian =  $(\exists x)(\exists y)(Ry \cdot Lxy)$

Some German loves some Russian =  $(\exists x)(Gx \cdot (\exists y)(Ry \cdot Lxy))$

Everyone loves some Russian =  $(x)(\exists y)(Ry \cdot Lxy)$

Every German loves some Russian =  $(x)(Gx \supset (\exists y)(Ry \cdot Lxy))$

Hans loves Olga = Lho

Hans loves everyone = ???

Hans loves every Russian = ???

Someone loves every Russian = ???

Some German loves every Russian = ???

Everyone loves every Russian = ???

Every German loves every Russian = ???

Hans loves Olga =  $Lho$

Hans loves everyone =  $(x)Lhx$

Hans loves every Russian =  $(x)(Rx \supset Lhx)$

Someone loves every Russian =  $(\exists x)(y)(Ry \supset Lxy)$

Some German loves every Russian =  $(\exists x)(Gx \cdot (y)(Ry \supset Lxy))$

Everyone loves every Russian =  $(x)(y)(Ry \supset Lxy)$

Every German loves every Russian =  $(x)(Gx \supset (y)(Ry \supset Lxy))$

	1	(x)Lxx	Valid
		[ ∴ (x)(∃y)Lxy	
*	2	asm: ∼(x)(∃y)Lxy	
*	3	∴ (∃x)∼(∃y)Lxy {from 2}	
*	4	∴ ∼(∃y)Lay {from 3}	
	5	∴ (y)∼Lay {from 4}	
	6	∴ ∼Laa {from 5}	
	7	∴ Laa {from 1}	
	8	∴ (x)(∃y)Lxy {from 2; 4 contradicts 6}	

Relational proofs are often tricky, even though they use no new inference rules. When you have a string of quantifiers, as in lines 2 and 3 above, work on one at a time, starting from the outside. Drop only *initial* quantifiers!

Invalid

- 1  $(x)Lxx$
- [  $\therefore (\exists x)(y)Lyx$
- \* 2 asm:  $\sim(\exists x)(y)Lyx$
- 3  $\therefore (x)\sim(y)Lyx$  {from 2}
- 4  $\therefore Laa$  {from 1}
- \* 5  $\therefore \sim(y)Lya$  {from 3}
- \* 6  $\therefore (\exists y)\sim Lya$  {from 5}
- 7  $\therefore \sim Lba$  {from 6}
- 8  $\therefore Lbb$  {from 1}
- \* 9  $\therefore \sim(y)Lyb$  {from 3}
- 10  $\therefore (\exists y)\sim Lyb$  {from 9} ...  $\rightarrow$  get c, d, ...

a, b

$Laa, Lbb$ $\sim Lba, \sim Lab$
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If you see an infinite loop coming, break out of it and invent your own refutation.
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# Endless Loops

Since everyone loves someone	a loves someone, call this person b b loves someone, call this person c c loves someone, call this person d ...
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$(x)(\exists y)Lxy$	$(\exists y)Lay \rightarrow Lab$ $(\exists y)Lby \rightarrow Lbc$ $(\exists y)Lcy \rightarrow Lcd$ ...
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# Alonzo Church's Theorem (1931)

The problem of determining validity in relational logic cannot be reduced to an algorithm (a finite mechanical procedure).

# Russell's theory of definite descriptions

The king of France is bald =  $(\exists x)((Kx \cdot \sim(\exists y)(\sim y=x \cdot Ky)) \cdot Bx)$

For some x,	x is a king of France and	no one else is a king of France	and x is bald
		there is no y such that y≠x and y is a king of France	

This symbolizes the English statement better than “Bk,” since:

- the statement can be false for three reasons (there's no king of France, there's more than one, or there's just one but with hair) and
- we more easily avoid the metaphysical error of thinking that “the round square” refers to an existing thing that isn't real.