Polar graph paper: http://incompetech.com/graphpaper/polar/

Suggested HW: 1, 2, 3, 4, 5, 6, 9, 12, ~~15, 16, 17, 18, 19, 21~~, 29, 31, 32, 34, 36, 37

www.jcu.edu/math "people" Stenson

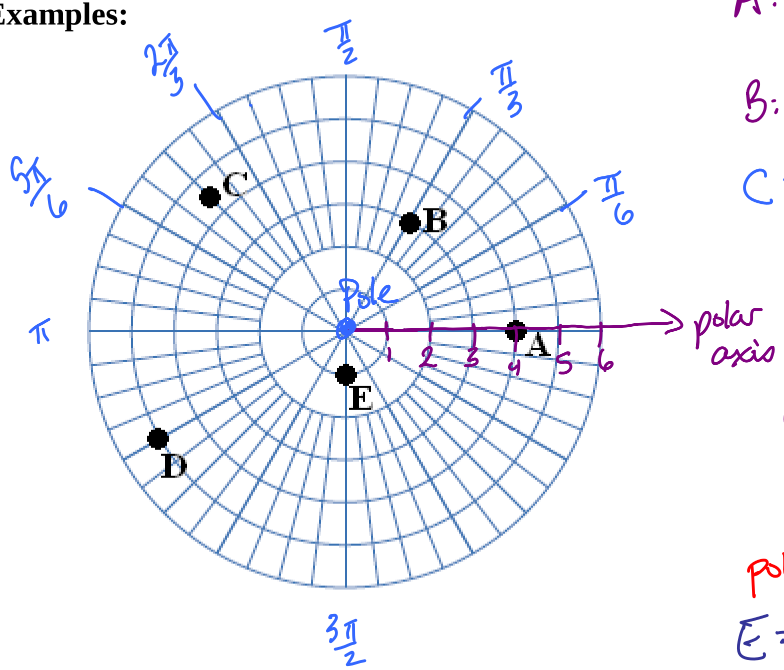
So far this year (and probably for most of the time you've been doing mathematics) you've considered points and graphs in the ***Cartesian coordinate system*** also known as the __rectangular__ coordinate system. For the most part, these coordinates work very well. But not all the time. So for the next several class periods, we're going to discuss another system: __polar__ coordinates.

The Basics

In the Cartesian coordinate system, we have a special point called the origin, and two axes (vertical and horizontal) that we use to locate points in the plane.

In polar coordinates, we have a special point (called the __pole__) and one horizontal ray (called the __polar axis__); this ray is horizontal with the endpoint at the pole, and extending infinitely to the right. The coordinates of a point have the form __(r, θ)__ , where $|r|$ is the __distance to the pole__ , and θ (in radians) is the angle through which the __polar axis__ must be rotated __counterclockwise__ to reach the point.

Examples:



A: $(4, 0)_p$

B: $(3, \frac{\pi}{3})_p$

C: $(4.5, \frac{3\pi}{4})_p$

D: $(5, \frac{7\pi}{6})_p$

E: $(1, \frac{3\pi}{2})_p = (1, -\frac{\pi}{2})_p$

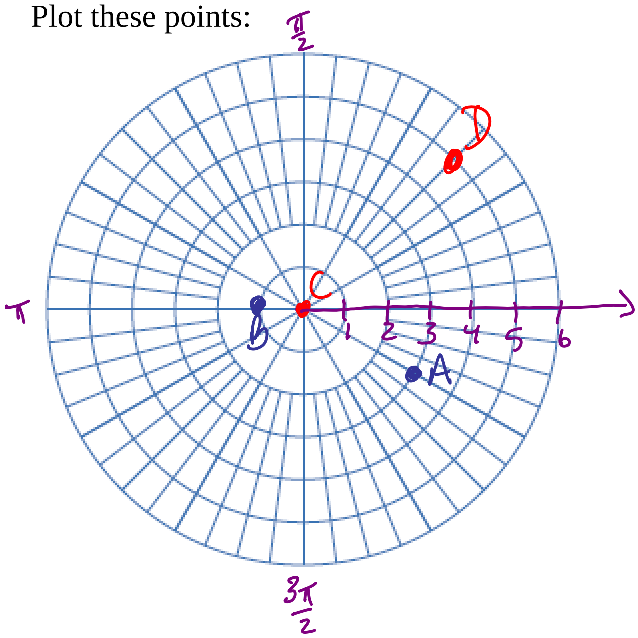
$= (1, -\frac{5\pi}{2})_p$

polar coordinates are not unique.

$E = (-1, \frac{\pi}{2})_p$

go -1 on polar axis; rotate $\frac{\pi}{2}$

Plot these points:



$$A = \left(3, -\frac{\pi}{6}\right)_P$$

$$B = (1, -3\pi)_P$$

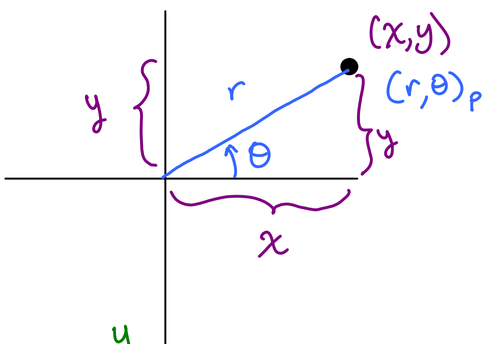
$$C = \left(0, \frac{3\pi}{4}\right)_P$$

$$D = \left(-5, \frac{13\pi}{4}\right)_P$$

$\frac{13\pi}{4} = \frac{12\pi}{4} + \frac{\pi}{4} = 3\pi + \frac{\pi}{4}$

Converting Between Cartesian and Polar Coordinates

Consider a point P with Cartesian coordinates (x, y) and polar coordinates (r, θ).



* $x = r\cos\theta$ $\quad \left(\cos\theta = \frac{x}{r}\right)$

* $x^2 + y^2 = r^2 \Rightarrow r = \pm\sqrt{x^2+y^2}$

* $y = r\sin\theta \Leftarrow \sin\theta = \frac{y}{r}$

If we know $(r,\theta)_P$, use

$x = r\cos\theta$, $y = r\sin\theta$

to find (x,y)

* $\tan\theta = \frac{y}{x}$

If we know (x,y), use

$\tan\theta = \frac{y}{x}$ and $x^2+y^2=r^2$ to find $(r,\theta)_P$.

Example: Find the Cartesian coordinates of the following points.

$$\left(2, -\frac{5\pi}{6}\right)_P$$

$x = r\cos\theta$, $y = r\sin\theta$

$x = 2\cos\left(-\frac{5\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

$y = 2\sin\left(-\frac{5\pi}{6}\right) = 2\left(-\frac{1}{2}\right) = -1$

So $\left(2, -\frac{5\pi}{6}\right)_p = (-\sqrt{3}, -1)_R$

$$(-1, 6\pi)_P$$

$x = -1\cos(6\pi) = -1(1) = -1$

$y = -1\sin(6\pi) = -1(0) = 0$

$(-1, 6\pi)_p = (-1, 0)_R$

Example: Find polar coordinates for each of the following points.

$r^2 = x^2 + y^2$; $\tan\theta = \frac{y}{x}$

$(6, 6\sqrt{3})$ ← Quad I

$\tan\theta = \frac{y}{x} = \frac{6\sqrt{3}}{6} = \sqrt{3}$

in $[0, 2\pi]$, $\theta = \frac{\pi}{3}, \frac{4\pi}{3}$

$r^2 = 6^2 + (6\sqrt{3})^2 = 144$

$r = \pm 12$

In Quadrant I: $\left(12, \frac{\pi}{3}\right)_p$ or $\left(-12, \frac{4\pi}{3}\right)_p$

$(-4, 4)$ ← Quadrant : II

$\tan\theta = \frac{4}{-4} = -1$; in $[0, 2\pi]$, $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$

$r^2 = (-4)^2 + 4^2 = 32$; $r = \pm\sqrt{32} = \pm 4\sqrt{2}$

In Quadrant II : $\left(\sqrt{32}, \frac{3\pi}{4}\right)_p$ $\left(-\sqrt{32}, \frac{7\pi}{4}\right)_p$

Graphing Polar Functions

In "polar" mode, your calculator can graph equations of the form $r = f(\theta)$. It's really, really important that you use a Square viewing window, so that circles look like circles.

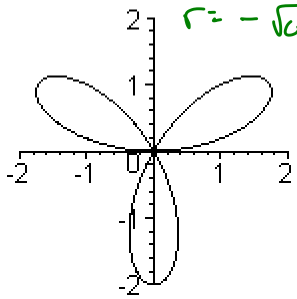
ZDECIMAL

Examples: Graph the following equations.

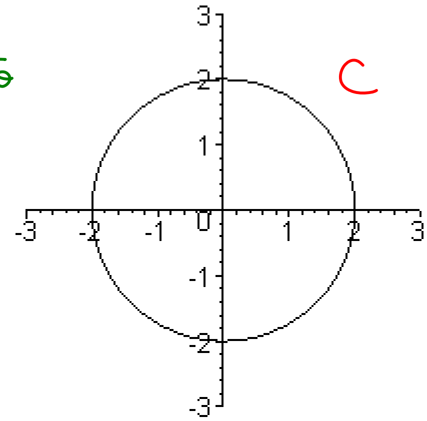
A. $r = 2\cos\theta$	B. $r = -3\sin\theta$	C. $r = 2$	D. $r = \cos(4\theta)$
E. $r = 2\sin(3\theta)$	F. $r = 1 + \cos x$	G. $r = \frac{3}{2} + \sin x$	H. $r = 1 - 2\sin x$ I
I. $r^2 = \cos 2\theta$	J. $r = \ln\theta$, $\theta \ge 1$	K. $r = \sec\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$	L. $\theta = \frac{\pi}{6}$

$r = \sqrt{\cos 2\theta}$

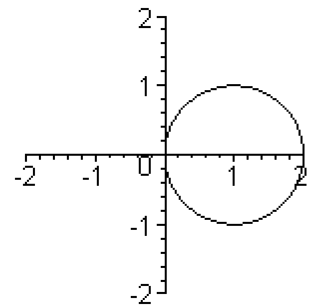
$r = -\sqrt{\cos 2\theta}$



C



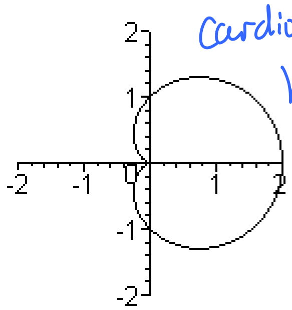
A



F

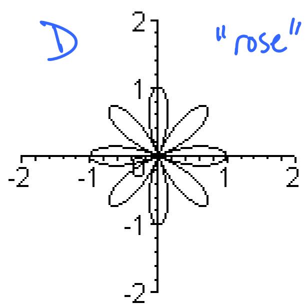
Cardiod

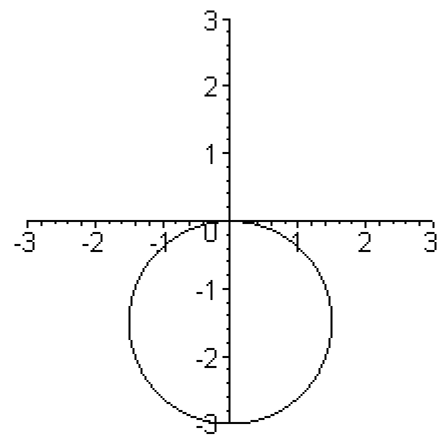
heart



D

"rose"

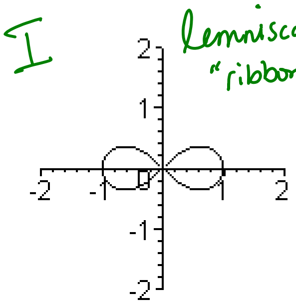




I

lemniscate

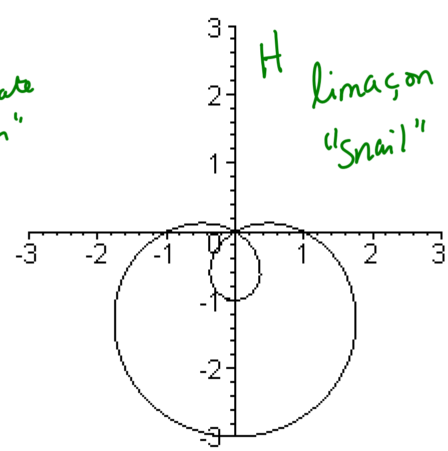
"ribbon"

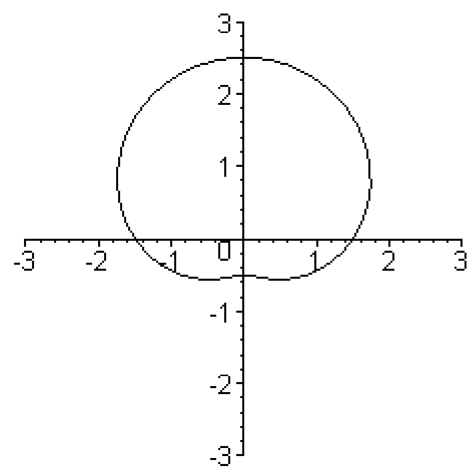


H

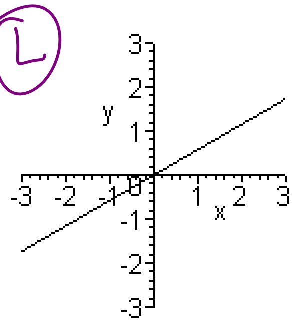
limaçon

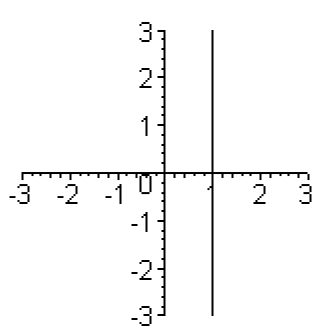
"snail"

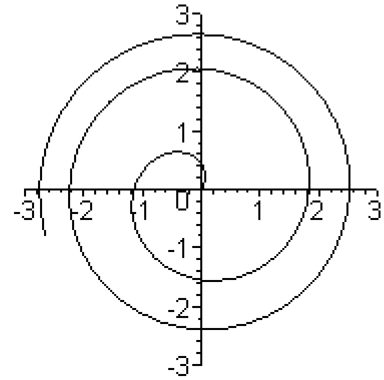




L







$$r = 3 - 3\sin\theta + \frac{\sin\theta\sqrt{|\cos\theta|}}{\sin\theta + 1.3}$$

Example: Write an equation in Cartesian coordinates that describes the curves given below in polar coordinates.

$$r\cos\theta = 8$$

$$r = 3\sin\theta$$

$$r^2 = \cos 2\theta$$

$$\theta = -\frac{\pi}{4}$$

Example: Write an equation in polar coordinates that describes the curves given below in Cartesian coordinates.

$$x^2 + y^2 = 16$$

$$y = 3x^2$$

Parametric Representation of Polar Curves

Any curve given by $r = f(\theta)$ can be parametrized in the Cartesian plane by

$x =$

$y =$

Slope

This brings us to the idea of slope which is a uniquely Cartesian concept. Still, if we have the graph of a polar equation, and the graph is smooth, we can draw tangent lines, and discuss the slope of those lines.

From our look at parametric equations, we know that if $r = f(\theta)$, then

$x =$

$y =$

So $\dfrac{dy}{dx} =$

Example: Find the slope of the line that is tangent to the graph of $r = 2\sin(3\theta)$, when $\theta = \frac{\pi}{3}$.