Example: Write an equation in Cartesian coordinates that describes the curves given below in polar coordinates.

$x = r\cos\theta$ | $x^2+y^2 = r^2$

$y = r\sin\theta$ | $\tan\theta = \frac{y}{x}$

$r\cos\theta = 8$

$x = 8$ (vertical line)

$r = 3\sin\theta$ ← multiply both sides by r

$r^2 = 3r\sin\theta$

$x^2+y^2 = 3y$ ← circle; complete the square

$x^2 + (y-\frac{3}{2})^2 = \frac{9}{4}$ (next section?) center at $(0, \frac{3}{2})$; radius $= \frac{3}{2}$

$r^2 = \cos 2\theta$

$r^2 = \cos^2\theta - \sin^2\theta$ ← mult. both sides by r^2

$r^4 = r^2\cos^2\theta - r^2\sin^2\theta$

$(x^2+y^2)^2 = x^2 - y^2$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= 1 - 2\sin^2\theta$

$= 2\cos^2\theta - 1$

$\theta = -\frac{\pi}{4}$

$\tan\theta = \frac{y}{x}$

$\tan(-\frac{\pi}{4}) = \frac{y}{x}$

$-1 = \frac{y}{x} \Rightarrow y = -x$

Example: Write an equation in polar coordinates that describes the curves given below in Cartesian coordinates.

$x^2 + y^2 = 16$

$r^2 = 16$

$\underbrace{r = 4 \text{ or } r = -4}_{\text{same graph in polar coords}}$

$x^2+y^2 = 16$ in polar coords is $r = 4$

$y = 3x^2$

$r\sin\theta = 3r^2\cos^2\theta$

$0 = 3r^2\cos^2\theta - r\sin\theta$

$0 = r(3r\cos^2\theta - \sin\theta)$

$r = 0$ or $3r\cos^2\theta - \sin\theta = 0$

$\boxed{r = \frac{\sin\theta}{3\cos^2\theta}}$ ← $\theta = 0, \pi$ makes $r = 0$

Don't need $r=0$ separate statement

Parametric Representation of Polar Curves

Any curve given by $r = f(\theta)$ can be parametrized in the Cartesian plane by

$x = r\cos\theta = f(\theta)\cos\theta$

$y = r\sin\theta = f(\theta)\sin\theta$

θ is the parameter

Slope

This brings us to the idea of slope which is a uniquely Cartesian concept. Still, if we have the graph of a polar equation, and the graph is smooth, we can draw tangent lines, and discuss the slope of those lines.

From our look at parametric equations, we know that if $r = f(\theta)$, then

$x =$

$y =$

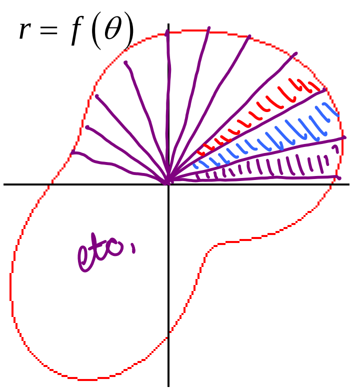
So $\dfrac{dy}{dx} =$

Example: Find the slope of the line that is tangent to the graph of $r = 2\sin(3\theta)$, when $\theta = \frac{\pi}{3}$.

Suggested HW: # 15, 16, 17, 18, 19, 21, 22, 24

(Section 10.3)

When finding area in the Cartesian coordinate system, we use rectangles as the basic shape. When we find area in the polar coordinate system, we use circles (slices of).



Each slice is a sector of a circle of radius r and central angle $\Delta\theta$

Area of sector ?

$$\frac{\text{Area of sector}}{\text{Area of whole}} = \frac{\Delta\theta}{\text{whole circle}}$$

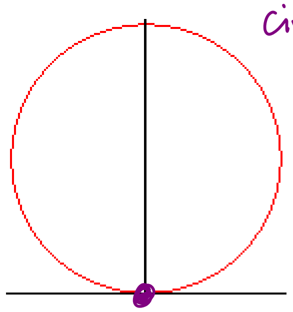
$$\frac{\text{Area of sector}}{\pi r^2} = \frac{\Delta\theta}{2\pi}$$

$$\text{Area of sector} = \pi r^2 \cdot \frac{\Delta\theta}{2\pi} = \frac{1}{2} r^2 \Delta\theta$$

$$= \frac{1}{2}[f(\theta)]^2 \Delta\theta$$

Example: Find the area enclosed by the graph of $r = 3\sin\theta$.

circle w/ radius $\frac{3}{2}$; $A = \frac{9}{4}\pi$



$$A = \int_0^{\pi} \frac{1}{2} r^2 \, d\theta = \int_0^{\pi} \frac{1}{2} (3\sin\theta)^2 \, d\theta$$

$$= \int_0^{\pi} \frac{1}{2} \cdot 9 \sin^2\theta \, d\theta \quad \leftarrow \text{half-angle identity}$$

$$= \frac{9}{2} \int_0^{\pi} \frac{1}{2}(1 - \cos 2\theta) \, d\theta$$

$$= \cdots = \frac{9}{4}\pi$$

$r = 0$

$3\sin\theta = 0$

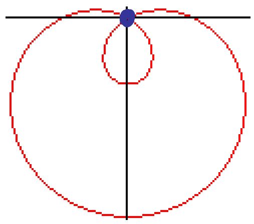
$\sin\theta = 0$

$\theta = 0, \pi, 2\pi$

check on calculator

if $\theta\text{min} = 0$, $\theta\text{max} = \pi$ } gives the right graph.

Example: Find the area inside the small loop of the graph of $r = 1 - 2\sin\theta$.



$$A = \int_{\pi/6}^{5\pi/6} \frac{1}{2} r^2 \, d\theta$$

$$= \int_{\pi/6}^{5\pi/6} \frac{1}{2}(1 - 2\sin\theta)^2 \, d\theta$$

limits of integration:
from one instance of $r=0$
to another.

$$\ldots = \pi - \frac{3\sqrt{3}}{2}$$

$1 - 2\sin\theta = 0$

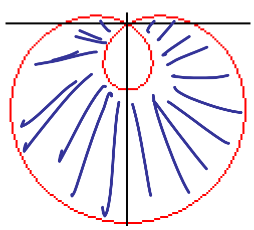
$1 = 2\sin\theta$

$\frac{1}{2} = \sin\theta$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$ Use calculator to determine which pair

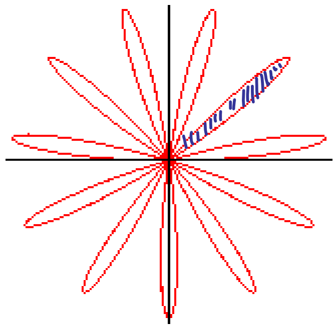
Example: Find the area inside the large loop, but outside the small loop of the graph of $r = 1 - 2\sin\theta$.

$\theta_{min} = \frac{5\pi}{6}$, $\theta_{max} = \frac{13\pi}{6}$: get the large loop.



$$A = \underbrace{\int_{5\pi/6}^{13\pi/6} \frac{1}{2}(1 - 2\sin\theta)^2 \, d\theta}_{\text{Area of region inside large loop}} - \int_{\pi/6}^{5\pi/6} \frac{1}{2}(1 - 2\sin\theta)^2 \, d\theta$$

Example: Find the area inside one leaf of the graph of $r = 3\sin(11\theta)$.



$r=0$ at $0 = 3\sin(11\theta)$

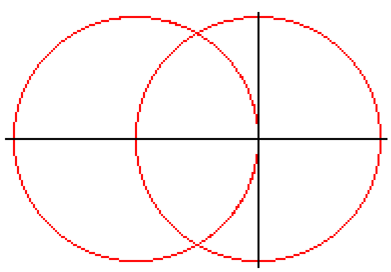
$11\theta = 0, \pi, 2\pi,$ etc.

$\theta = 0, \frac{\pi}{11}, \frac{2\pi}{11},$ etc.

$$A = \int_0^{\frac{\pi}{11}} \frac{1}{2}\left(3\sin(11\theta)\right)^2 d\theta$$

$$= \dots = \frac{9\pi}{44}$$

Example: Find the area of the region inside the graph of $r = -4\cos\theta$ and outside the graph of $r = 2$.



Suggested Homework: #5, 6, 8, 9, 12, 17, 20, 23, 24, 27, 30, 36