Introduction to LiveMath

Materials for Minicourse SAT-M13

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INTRODUCTION TO LIVEMATH

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Interactive mathematics activities allow students to experiment with mathematical concepts. Students can form conjectures about general results, or simply check that they are doing the steps of a problem correctly. Such interactive activities can be implemented in a variety of settings, such as graphing calculators and computer algebra systems. In this paper, we discuss using the LiveMath Maker software to create interactive web pages. Students can access the pages from most internet browsers with the appropriate free plug-in, and are therefore not restricted to using campus computer labs. Moreover, web pages generally allow students to see all the relevant information on one screen – a feature that cannot be duplicated on a graphing calculator.

We like LiveMath pages because students can make changes to variables or functions and immediately see the results of those changes:

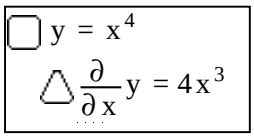


Figure 1:
Before the change

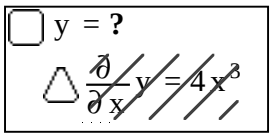


Figure 2:
During the change

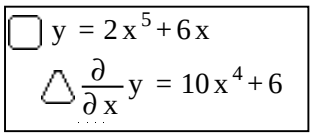


Figure 3:
After the change

Samples of some web pages that include LiveMath notebooks may be found on our web site: `http://www.jcu.edu/math/ictcm2001/`.

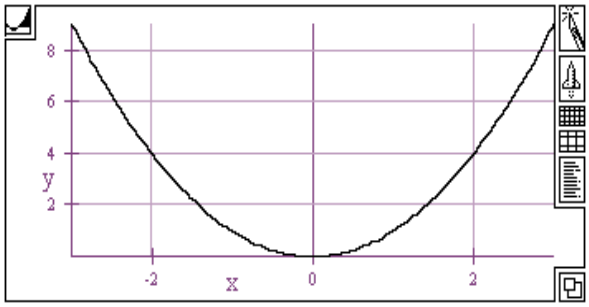
In the following pages, we provide step-by-step instructions for using LiveMath Maker to create some LiveMath notebooks. Once you create a notebook in LiveMath, select "Save as Plug-In with HTML..." from the File menu. The result will be two files: a LiveMath file (with the .thp extension) and an HTML document containing a tag that accesses the LiveMath document. The web page will not function without the LiveMath file, so you must put both files on your web server when you are ready for students to access them. You can edit the web page (the HTML document) using any web-authoring tool. (We use Netscape Composer, which comes free with Netscape Communicator.)

Basic Graphing

Enter an equation such as $y = x^2$, using the ^ key to enter the exponent. LiveMath will display the equation in standard mathematical notation. (You can use other variables, but if you do, LiveMath may ask you to confirm your intended meanings for these variables.)

$y = x^2$

Highlight the equation by clicking on its equal sign, and then click the 2D graph button in the Graph section of the palette . Alternatively, choose Graph … Graph 2D … Curve.



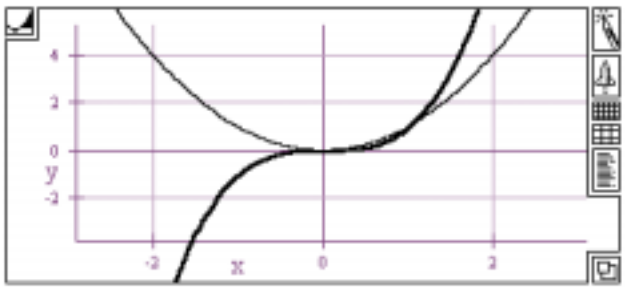
The resulting graph is highly interactive:

- You can move the viewing window by clicking and dragging the "paper" on which the graph is drawn.
- You can resize the graph by dragging the lower right corner .
- You can zoom out with the rocket ship button , and zoom in with Ctrl-Alt- .
- The knife button allows you to zoom with a zoom box.
- The two buttons that look like grid lines control the resolution of the graph.
- You can change various graph attributes such as color, line style, and window boundaries, by clicking the button, which opens the "graph details" box.

If you wish to graph a second function on the same coordinate system, you must define it with a new independent variable, such as y_2. (Use the underscore to get the subscript.)

$y_2 = x^3$

Then highlight this new equation, and choose Graph … Graph 2D … Add Curve Plot.



The Quadratic Formula

Type in three lines to define the initial values of the coefficients in a quadratic equation.

$a = 2$
$b = 3$
$c = 1$

Now type in the quadratic equation $ax^2 + bx + c = 0$, using * for multiplication. (It won't show!) Then Shift-click on the equal signs for *a*, *b* and *c*, so that all three equations are highlighted. Then hold down Ctrl and drag all three equations over the quadratic equation, so that the square to the left and all three coefficients in the equation are highlighted. Let go. (The conclusion is moved over by clicking Notebook… Formatting…Indent Right—or by Ctrl-dragging it.)

$a x^2 + b x + c = 0$ $2x^2 + 3x + 1 = 0$ Substitute

Now type in the formula for the first root, using **sqrt** for the square root. Be sure to use parentheses around the numerator and around the denominator. (They will disappear later.)

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

To prevent LiveMath from doing too much too fast, toggle off the "Auto Simplify" feature by pressing the lower left button on the "Compute" palette  or by selecting Compute…Computation Prefs and unchecking Auto Simplify. Then Shift-click the equations for *a*, *b* and *c*, and Ctrl-drag them into the formula for the root.

$$x = \frac{-3 + \sqrt{3^2 - 4 \cdot 2 \cdot 1}}{2 \cdot 2}$$ Substitute

Now highlight *only the part under the radical*. Then Shift-drag to also highlight the denominator. Then click the Simplify button in the palette  or select Compute… Simplify:

$$x = \frac{-3 + \sqrt{1}}{4}$$ Simplify Simplify

Finally, highlight this most recent expression, and click the Simplify button:

$$x = -\frac{1}{2}$$ Simplify

Repeat this process with the second root of the equation.

You may wish to conceal the steps in the computations of the roots, and encourage students to try the work for themselves before clicking to reveal the solution. To do this, click the ▢ in front of the original expression for the root, and select Notebook… Formatting…Collapse, and save the notebook with these steps collapsed. You can open and collapse the remaining steps by double-clicking on ▢ .

Completing the Square

This LiveMath notebook displays the steps involved with completing the square in a quadratic polynomial with leading coefficient $a = 1$. To begin, type in two lines to define the initial values of the other two coefficients, *b* and *c*:

$b := 8$
$c := 4$

Now type in the first step in the process of completing the square. Remember to use * for multiplication in the "b x." (The asterisk will not show.)

$$x^2 + b\,x + c = \left(x^2 + b\,x + \left[\frac{b}{2}\right]^2\right) - \left(\frac{b}{2}\right)^2 + c$$

Substitute b and c only into the left side of this expression, by Shift-clicking on the equal signs for b and c, and then Ctrl-dragging them to the left side of the equation:

$$x^2 + 8\,x + 4 = \left(x^2 + b\,x + \left[\frac{b}{2}\right]^2\right) - \left(\frac{b}{2}\right)^2 + c \quad \text{Substitute}$$

Now highlight *only the part in the first set of parentheses* on the right, and choose Compute…Factor or click the Factor button in the palette:

$$x^2 + 8\,x + 4 = \left(x + \tfrac{1}{2}b\right)^2 - \left(\frac{b}{2}\right)^2 + c \quad \text{Factor}$$

If LiveMath factors this expression as $(\frac{1}{2}b + x)^2$ instead of $(x + \frac{1}{2}b)^2$, you may wish to highlight just the $\frac{1}{2}b + x$ and choose Compute…Commute.

Next drag the value of *b* onto the right hand side:

$$x^2 + 8\,x + 4 = (x + 4)^2 + c - 16 \quad \text{Substitute}$$

Finally, highlight this last equation and choose Compute…Calculate:

$$x^2 + 8\,x + 4 = (x + 4)^2 - 12 \quad \text{Calculate}$$

You may wish to conceal the steps in the computations, and encourage students to try the work for themselves before clicking to reveal the solution. To do this, click the ▢ in front of the original expression for the quadratic, and select Notebook…Formatting…Collapse, and save the notebook with these steps collapsed. You can open and collapse the remaining steps by double-clicking on ▢. Remember that if you intend to post this page to the web, you will need to increase the height attribute of the embedded page, to allow for the increased size of the page when the calculations are visible.

Translations and Reflections

Enter values for *a*, *h*, and *k*. Make each statement a "working statement" by highlighting it and pressing the button that is fourth from the left on the bottom row of the Compute palette: . Aternately, highlight each statement, and select Working Statements ... Make Working Stmt from the Compute menu.

$a = 1$
$h = 0$
$k = 0$

You can put these statements on the same line of the workbook by clicking the to highlight the statement, and then holding down Ctrl while you drag the statement to the location you want. Don't let go until you see a darkened rectangle more or less where you want the statement to go.

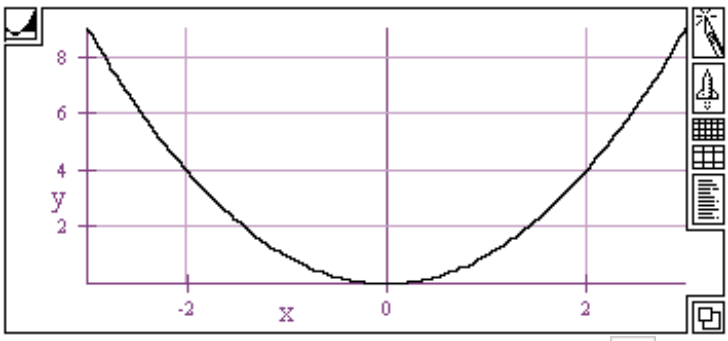
Enter the function for *y*, using the variables *a*, *h*, and *k*. Use the carat key to get exponents, and use the right cursor key to get out of the exponent position.

$y = a(x - h)^2 + k$

Substitute the values for *a*, *h*, and *k* into the expression for *y*: Click on the equal sign in definition for *a*, and shift-click on the equal sign in definitions for *h* and *k*. Then hold down Ctrl and drag the expressions over the definition for *y* until the square at the left is darkened. Let go.

$y = x^2$ Substitute

Highlight the version of the function that contains the letters *a*, *h*, and *k*, and graph it by pressing the button in the upper left corner of the Graph palette: . Alternately, select Graph2D ... Curve from the Graph menu. Accept the variables that LiveMath suggests. If necessary, click on the graph region to draw the graph.



Now change the way the graph looks, if you so choose. Click on the graph details icon. (It's the fifth button from the top at the right side of the graph window: .) You can change the dimensions of the window here by changing the numbers that are defined to be "left," "right," "top," and "bottom." Double click on the Declarations icon in the Graph Details window: . If you want more grid lines, change the "separated by 0" statement in the two "Grid lines" sentences to "separated by 1". You can change the colors of the grid lines in those statements as well. You can change the colors of the axes in the two "Axis" statements. Change the location of the *x*- and *y*-axes by changing the words "left" and "bottom" in the "Axis" statements to 0. Close the Declarations and Graph Details boxes by clicking on the respective icons twice (resp. once).

Simplifying the Difference Quotient

To prevent LiveMath from simplifying too much too fast, begin by toggling off the "Auto Simplify" feature: depress the button in the lower left of the "Compute" palette. 

Define the function *f*, using wildcard variables. Get wildcard variables by clicking on the button in the Symbol palette that looks like this:  . From the submenu, choose "x." Alternatively, type a question mark (?) followed by the letter *x*. Use the carat key to get exponents, and use the right cursor key to move out of the exponent.

$$f(x) = x^2 + 3x$$

Clarify the notebook (under the Notebook menu) to define *f* as a function. Then enter the difference quotient. Type "f(x + h) - f(x)", then highlight it and hit the "/" key to get a fraction form.

$$\frac{f(x+h) - f(x)}{h}$$

Substitute the expression for f(x) into the difference quotient by clicking once on the equal sign in the f(x) statement to highlight. Then hold down Ctrl and drag the f(x) expression over the difference quotient expression until both f(x + h) and f(x) are darkened. Then let go.

$$\frac{f(x+h) - f(x)}{h} = \frac{([x+h]^2 + 3[x+h]) - (x^2 + 3x)}{h} \quad \text{Substitute}$$

Expand the numerator by highlighting the numerator and choosing "Expand" from the Compute menu.

$$\frac{f(x+h) - f(x)}{h} = \frac{([h^2 + x^2 + 2hx] + [3h + 3x]) - (x^2 + 3x)}{h} \quad \text{Expand}$$

Simplify the numerator by highlighting it and choosing "Simplify" from the Compute menu.

$$\frac{f(x+h) - f(x)}{h} = \frac{h^2 + 3h + 2hx}{h} \quad \text{Simplify}$$

Factor the numerator by highlighting it and choosing "Collect" from the Compute menu.

$$\frac{f(x+h) - f(x)}{h} = \frac{h(1h + [2x + 3])}{h} \quad \text{Collect}$$

Now highlight the entire right hand side and choose "Simplify" from the Calculate menu.

$$\frac{f(x+h) - f(x)}{h} = h + 2x + 3 \quad \text{Simplify}$$

Exploring the Product Rule

Type in functions f(x) and g(x). Then enter p(x) as the product of *f* and *g*. (It's natural to want to use the letter *h* for that third function, but don't. The letter *h* is pre-defined as a variable, and not as a function.) This page will work better if at least one of f(x) and g(x) is not a polynomial.

$f(x) = 3x^2 + 4x$

$g(x) = \sin(x)$

$p(x) = f(x)\,g(x)$

Clarify the notebook (under the Notebook menu) to define *f*, *g*, and *p* as functions.

Substitute the expressions for f(x) and g(x) into p(x): Click on the equal sign in the f(x) definition, and shift-click on the equal sign in the g(x) definition. Then hold down Ctrl and drag the two expressions over the p(x) definition until the square at the left is darkened. Let go.

$p(x) = (3x^2 + 4x)\sin(x)$ Substitute

On the next blank line click the derivative button from the "Build" palette:  . Type "f(x)" (without the quote marks) and then highlight the lower question mark and type "x."

$\frac{\partial}{\partial x} f(x)$

Now substitute the expression for f(x) by clicking on the equal sign in the f(x) definition and dragging it to the derivative expression.

$\frac{\partial}{\partial x} f(x) = 6x + 4$ Substitute

Repeat this process to get the derivatives of g(x) and p(x). (When you substitute p(x), don't use the original p(x) definition with *f*'s and *g*'s.) If you don't like the position of any of these lines, you can change their indentation under the Notebook ... Formatting menu.

$\frac{\partial}{\partial x} g(x)$

$\frac{\partial}{\partial x} g(x) = \cos(x)$ Substitute

$\frac{\partial}{\partial x} p(x)$

$\frac{\partial}{\partial x} p(x) = (3x^2 + 4x)\cos(x) + (6x + 4)\sin(x)$ Substitute

Linear Transformations

Enter a matrix (called *square*) containing the points (0, 0), (0, 1), (1, 1), (1, 0), and (0, 0) as column vectors. These are the vertices of a square which we will draw in LiveMath. The origin is listed twice, as it is both the starting point and the ending point. To enter this matrix, type the following (without the quote marks): "square = 0,0,1,1,0;0,1,1,0,0". Note the semicolon that indicates a new row in the matrix. As you type, parentheses and question marks will appear on the screen. Ignore them. When you finish, you should see a matrix as shown here:

$$\text{square} = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

Define a 2×2 matrix *A* that represents a linear transformation. Type "A=1,0;0,1".

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Enter the equation *B = A * square*. You must use the asterisk for multiplication, even though the asterisk does not appear on the screen.

$$B = A \cdot \text{square}$$

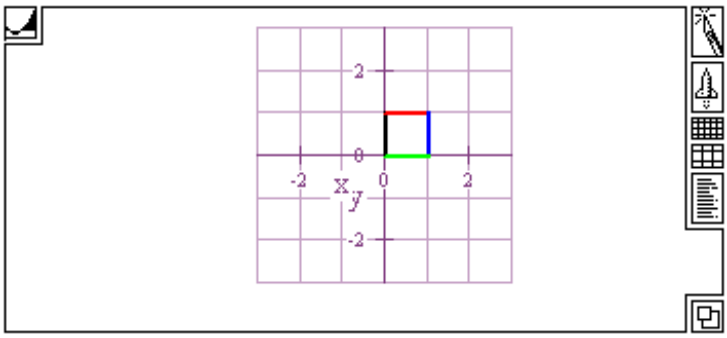
Clarify the notebook under the Notebook menu. Accept LiveMath's suggestion that *square* and *A* be matrices. For *B*, change "variable" to "matrix."

To draw the sides of the square, we need a graph window with space for four different lines. Enter the dummy equation $y = x$. Highlight the equation and plot the graph by pressing the button in the upper left corner of the Graph palette:  . From the Graph menu, select Graph2D...Add Curve Plot. Do this a total of three times. The result will be four copies of the line $y = x$ (in different colors) superimposed on one another.

Click on the Graph Details button at the right of the graphing window:  . Change "stretch to fit" to "true proportions." Change the four "curve at" statements to match the statements below. Use the underscore key to move into subscript position.

Curve at $\left(B_{[1,\,u]}, B_{[2,\,u]}\right)$ where $u = 1 \ldots 2$ with a heavy line, colored Black.
Curve at $\left(B_{[1,\,u]}, B_{[2,\,u]}\right)$ where $u = 2 \ldots 3$ with a heavy line, colored Red.
Curve at $\left(B_{[1,\,u]}, B_{[2,\,u]}\right)$ where $u = 3 \ldots 4$ with a heavy line, colored Blue.
Curve at $\left(B_{[1,\,u]}, B_{[2,\,u]}\right)$ where $u = 4 \ldots 5$ with a heavy line, colored Green.

Do more formatting in the Declarations section if you wish. Start by double clicking the Declarations text bubble in the Graph Details window: . For more grid lines on the graph, change the "separated by 0" statement in the two "Grid lines" sentences to "separated by 1". Change the location of the *x*- and *y*-axes by changing the words "left" and "bottom" in the "Axis" statements to 0. When you are finished, close the Declarations and Graph Details boxes by clicking on the respective icons twice (resp. once).



Students can now make changes to the linear transformation *A* and see the resulting changes in the square.