

The purpose of these exercises is to explore functions that are defined using definite integrals.

### Definite Integrals in Derive:

To compute a definite integral, choose “Integrate” from the Calculus menu. In the “Integrate” dialog box (see Figure 1), choose the variable of integration from the drop-down menu and make sure the integral type is “Definite.” Type the upper and lower limits of integration in the appropriate fields. Then click “OK,” followed by a “Basic” simplification from the Simplify menu.

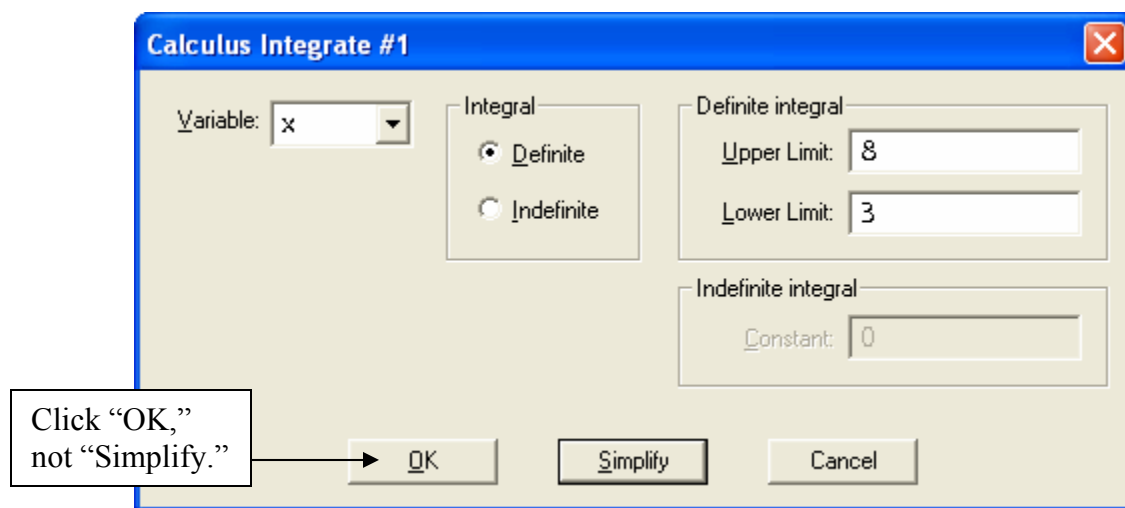


Figure 1: “Integrate” Dialog Box

### Try This:

Author the expression  $x\sqrt{3x^2+4}$ , and compute  $\int_3^8 x\sqrt{3x^2+4} dx$ . Click “OK” in the “Integrate” dialog box. Verify that you’ve entered the correct information, and then simplify. The result should be  $\frac{2744}{9} - \frac{31\sqrt{31}}{9}$ .

*I will hold office hours in the computer lab in Dolan E223 on Monday, November 8, from 8:00 until 9:00 p.m., and on Tuesday, November 9, from 11:00 a.m. until noon. If you would like help at another time, PLEASE come see me.*

**Exercises:**

Use *Derive*<sup>TM</sup> 6 for the following exercises. Use standard mathematical notation to record the results *on a separate sheet of notebook paper*. Do not turn in a print-out of your Derive session.

- For each function  $F(x)$  below, do the following. (See the example on the following page.)
  - Use the Calculus ... Integrate feature to enter the integral expression for  $F(x)$  into Derive.
  - Find an exact value of  $F(-2)$ ,  $F(0)$ , and  $F(2)$ . If Derive doesn't return a numerical value, or it looks "strange," then give an approximation with four decimal places of precision. On your paper, show the integral form of each of these numbers, along with the numerical value. (For example,  $F(-4) = \int_0^{-4} t^3 dt = 64$ .)
  - Find (and simplify)  $F'(x)$ , by using the Calculus ... Differentiate feature of Derive.

(a)  $F(x) = \int_0^x (2t + 10) dt$

(b)  $F(x) = \int_2^x (2t + 10) dt$

(c)  $F(x) = \int_0^x \frac{1}{\sqrt{t^4 + 1}} dt$

(d)  $F(x) = \int_{-2}^x \frac{1}{\sqrt{t^4 + 1}} dt$

(e)  $F(x) = \int_0^x e^{t^3} dt$  (Be sure to use the constant  $e$  and not the variable  $e$ .)

(f)  $F(x) = \int_2^x e^{t^3} dt$

- Each function  $F(x)$  in Question 1 has the form  $F(x) = \int_a^x g(t) dt$ . For each function in Question 1, identify the value of  $a$  and the function  $g(t)$ .

- Complete the following sentence: Based on the results of Question 1, I believe that if  $F(x) = \int_0^x g(t) dt$ , then  $F'(x) =$  \_\_\_\_\_.

- Differentiate these functions, using your formula from Question 3. (SHOW ME!) Then differentiate these functions with Derive, and determine whether your formula from Question 3 gives the correct derivatives. Derive may perform some simplification, so analyze your results carefully.

(a)  $F(x) = \int_1^x \ln(e^t + t) dt$

(b)  $F(x) = \int_3^x \ln(e^t + t) dt$

(c)  $F(x) = \int_0^x t(3t^2 + 1) dt$

(d)  $F(x) = \int_{-2}^x t(3t^2 + 1) dt$

**Example:**

$$\text{Let } F(x) = \int_2^x (t^2 + 16)^{-1/3} dt$$

To find  $F(-2) = \int_2^{-2} (t^2 + 16)^{-1/3} dt$ , we author the expression  $(t^2 + 16)^{-1/3}$ . Then we use the Calculus ... Integrate feature to build the expression  $\int_2^{-2} (t^2 + 16)^{-1/3} dt$ . Finally, we simplify the integral expression to get  $-2 \int_0^2 \frac{1}{(t^2 + 16)^{1/3}} dt$ . Since this isn't a recognizable numerical form, we approximate it.

**Write this on your paper:**

$$F(x) = \int_2^x (t^2 + 16)^{-1/3} dt$$

$$F(-2) = \int_2^{-2} (t^2 + 16)^{-1/3} dt \approx -1.547$$

To find  $F'(x)$ , we author the expression  $(t^2 + 16)^{-1/3}$ . Then we use the Calculus ... Integrate feature to build the expression  $\int_2^x (t^2 + 16)^{-1/3} dt$ . With this last expression highlighted, we use the Calculus ... Differentiate feature to build the expression  $\frac{d}{dx} \int_2^x (t^2 + 16)^{-1/3} dt$ . Finally, we simplify.

**Write this on your paper:**

$$F(x) = \int_2^x (t^2 + 16)^{-1/3} dt$$

$$F'(x) = \frac{1}{(x^2 + 16)^{1/3}}$$