USING *DERIVE*™ *6* IN LIBERAL ARTS/BUSINESS CALCULUS

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For years, *Derive* has been an excellent supplement to a Liberal Arts/Business Calculus course, in part because of its ease of use compared to other computer algebra systems. Some of the new features in *Derive 6* open additional opportunities for classroom demonstrations and student learning. Among these new features are

- slider bars in 2D and 3D graphs
- the "display steps" feature
- communication with certain TI handheld devices.

In our conference paper, we discuss some of the specific applications of *Derive* that we are currently using in teaching our respective calculus classes. In this handout, we focus on the use of slider bars in classroom demonstrations, and also provide a sample of a discovery learning activity that utilizes student investigations with *Derive*.

Tangent Line Demonstration

Watching a sequence of secant lines "become" a tangent line is a powerful introduction to the derivative as a limit. Slider bars allow us to seamlessly move one of the defining points of a secant line along a curve until it is extremely close to the other defining point, thus creating a line that is essentially a tangent line.

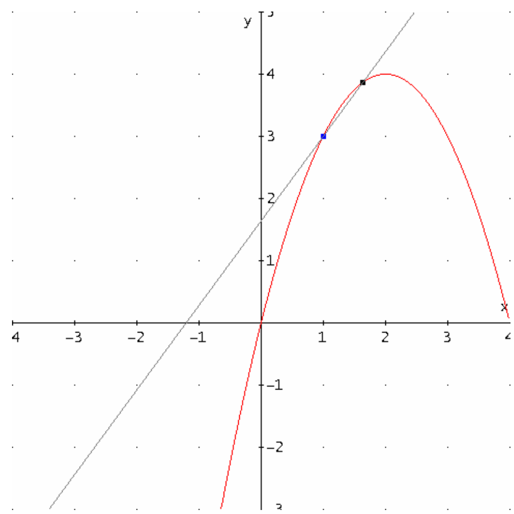
Before going to class, we create a worksheet containing the following:

- A defined function $f(x)$ for which we will draw secant lines.
- The defining points of the secant line: $\left[a, f(a)\right]$ and $\left[a+\frac{h}{50}, f\left(a+\frac{h}{50}\right)\right]$. When we create slider bars in the plot window, the minimum and maximum values of *h* must be integers. Since $h=0$ causes our secant line to disappear, we use $h=1$ as our minimum value, and use $h/50$ instead of just *h* in our defining equations.
- A defined "slope" function $m(a,h)=\dfrac{f\left(a+\frac{h}{50}\right)-f(a)}{\frac{h}{50}}$ that will give the slope of the secant line. The equation of the secant line: $y-f(a)=m(a,h)(x-a)$.

Since we can't save plot settings in *Derive*, we have to set up the plot window when we get to class and open the secant lines worksheet we saved previously:

- Select "large" points from the Options ... Display ... Points menu.
- Set an appropriate plot range for our function and set the aspect ratio to 1:1.

- Insert the slider bars for *a* and *h*. The maximum number of intervals for a slider bar is 50. We use a relatively small number of intervals for *a*. In order to have the final secant line in the sequence look essentially like the tangent line, the distance between the two defining points must be quite small. So we set the bar for *h* so that $1 \le h \le 51$ with a large number of intervals.
- Use the slider bars to set the values of *a* and *h* to be used for the initial plot. The value of *a* should reflect the point of tangency, and *h* should be as large as possible.
- Plot $f(x)$ and the two points. Highlight the equation for the secant line, so that it's ready to plot when the demonstration begins.



- Minimize the algebra window or hide it behind the plot window, unless we want to explain the algebra to our students.

When we're ready to show the demonstration to the students, we plot the secant line. Then we move the slider for *h* and discuss the results. If we feel the need, we reset *h* at its maximum value, and move the slider for *a* to another value.

Student "Discovery" of Differentiation Theorems

We have had some success guiding students to discover derivative results such as the power and product rules using *Derive*. Students gain valuable experience in forming and testing conjectures, and our hope is that students will more easily remember results that they feel they had a hand in creating. As an example, we describe below an activity involving the power rule.

After using the limit definition of derivative to compute the derivative of x^2 by hand, we give students a list of power functions with larger positive integer exponents. For each function, we ask them to use *Derive* to first simplify the difference quotient, and then to compute the limit of the difference quotient as *h* approaches 0. Based on their results, we ask them to conjecture a formula for the derivative of x^n, where *n* is a positive integer. After they test their conjecture for several other positive values of *n*, we ask them to test it for some negative integer exponents. *Derive* returns expressions like $-1/x^2$ instead of $-x^{-2}$, so students may at first believe that their formula is valid only for positive exponents. Some students go so far as to formulate a new conjecture for negative exponents. This provides us a valuable opportunity to stress the importance of being able to change between different algebraic representations of the same quantity. If time permits, we ask students to test their conjectures for rational exponents as well.

One reason we particularly appreciate the power rule activity is that it reinforces the students' experience with the derivative as a limit, and they see *Derive* as an expedient for performing algebraic simplification, rather than as a "black box" for computing derivatives based on secret formulas. A copy of our Power Rule discovery activity is included on the next page.

MT 135 **Derivatives with *Derive*^{TM} *6*, Part I** **Fall, 2004**

The purpose of these exercises is to compute the derivatives of functions of the form $f(x) = x^n$, where n is an integer.

Directions:
Use *Derive*^{TM} *6* for the following exercises. Use standard mathematical notation to record the results ***on a separate sheet of notebook paper.*** Do not turn in a print-out of your Derive session.

For each function in the exercises below, do the following:

(a) Author the expression $\dfrac{f(x+h)-f(x)}{h}$ for the specific function $f(x)$.

(b) Simplify the expression $\dfrac{f(x+h)-f(x)}{h}$. (Notice that this simplification is valid only if $h \neq 0$.)

(c) Compute $\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$, which is $f'(x)$.

Copying Results from Derive onto Your Homework Paper:
What you record on your homework paper should look something like this. That is, you should record more than just the end result from Derive.

$$f(x) = x^2$$
$$\frac{f(x+h)-f(x)}{h} = \frac{(x+h)^2 - x^2}{h} = 2x + h$$
$$f'(x) = \lim_{h\to 0}(2x+h) = 2x$$

> You need to author $\dfrac{(x+h)^2 - x^2}{h}$. Then have Derive simplify to get $2x+h$.

Exercises

1. $f(x) = x^3$ 2. $f(x) = x^4$ 3. $f(x) = x^5$ 4. $f(x) = x^6$

Complete the following sentence: Based on my results in Exercises 1-4, I believe that if n is a positive integer and $f(x) = x^n$, then $f'(x) =$ ____________________ .

Does your formula work when $n = 2$? (See the example above.) Does your formula work when $n = 1$? Use Derive to find out!

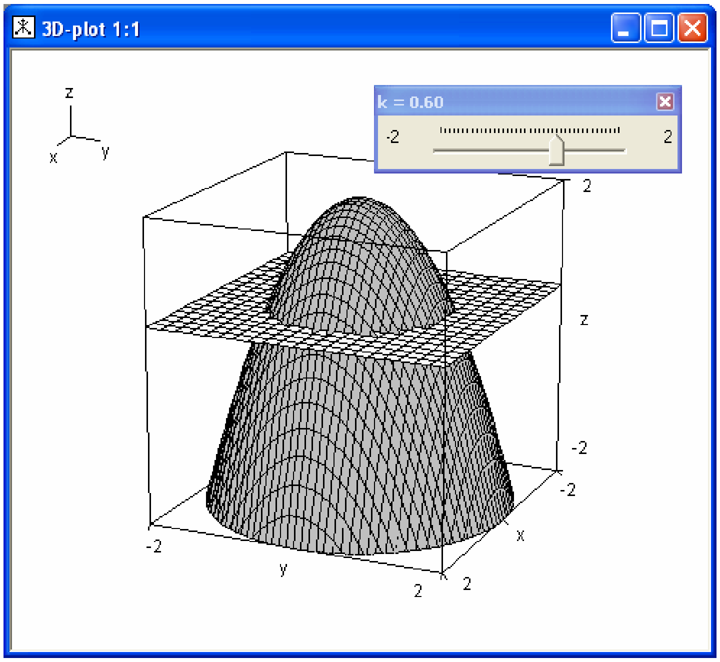
For the functions below, use your formula to find $f'(x)$. Then follow steps (a)-(c) above to have Derive compute $f'(x)$.

5. $f(x) = x^{-1}$ 6. $f(x) = x^{-2}$ 7. $f(x) = x^{-3}$

Does your formula for $f'(x)$ seem to work when n is a negative integer?

Multivariable Functions and Slider Bars

Students in the Liberal Arts/Business Calculus course often have difficulty visualizing graphs in three dimensions. *Derive*'s 3D plotting capabilities, along with the use of slider bars, can help students to understand the nature of graphs of functions of two variables. For instance, to demonstrate the idea of level curves, we can plot the graph of a simple function $z = f(x, y)$, along with the graph of the plane $z = k$. By attaching the variable k to a slider bar, we can create a simple but effective demonstration of the level curves for the surface. *Derive*'s ability to rotate the graph in real time assures that students are able to understand the graph, since they can see it from a variety of viewpoints. The following picture shows this for the graph of $z = 2 - x^2 - y^2$.



Copies of our conference paper and the *Derive* activities we have used in our classes are available online at our web site: http://www.jcu.edu/math/ICTCM2004. A free 30-day trial copy of *Derive* is available from Texas Instruments, at http://education.ti.com/derive.