MT 135-56/8 **More Algebra with *Derive*^{TM} *6*** **Fall, 2004**

In the previous activity, you learned how to use Derive to do some basic algebraic manipulations. The purpose of this worksheet is to help you expand your abilities with Derive.

Getting Started:

Run *Derive*^{TM} *6* from any of the computer labs on campus. If the algebra window doesn't fill the entire Derive screen, click the "Maximize" button at the top right corner of the algebra window.

Using the Symbol Toolbars:

There are actually two symbol toolbars side-by-side at the bottom of the Derive screen. One toolbar contains upper and lower case Greek letters. The other toolbar contains mathematical symbols. See Figures 1 and 2 below.

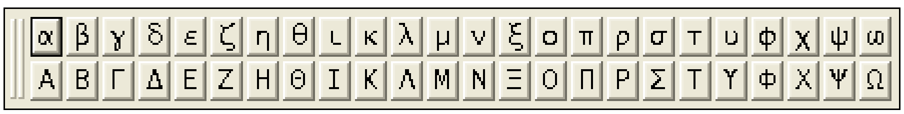

Figure 1: Greek Symbol Toolbar

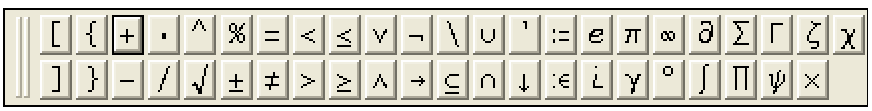

Figure 2: Math Symbol Toolbar

Notice that some symbols (like π) show up on both toolbars. Clicking the π button on the Greek symbol tool bar is **NOT** the same as clicking the π on the math symbol keyboard! The Greek letters from the Greek symbol toolbar can be used as variables in Derive. Most of them can also be accessed by typing their name in the expression entry line. Type "beta" to get the variable β, for example. The exceptions are the Greek letters that have common mathematical meanings. You can tell which letters these are, because they show up again on the math symbol toolbar. If you type "pi," you get the mathematical constant $\pi = 3.14159...$ and not a variable named π.

Try This:

Author the expression $2\theta = \frac{\pi}{3}$, where θ is a variable and π is the usual constant. Then solve the equation. You should see $\theta = \frac{\pi}{6}$ as your result.

Numerical Approximations:
We will mostly be using Derive to give exact values of computations, but we will occasionally want a numerical approximation. To get an approximation, highlight the desired expression (you may need to author it first). Then choose "Approximate" from the Simplify menu. In the dialog box (see Figure 3), you can indicate the desired number of decimal places. Then click "Approximate."

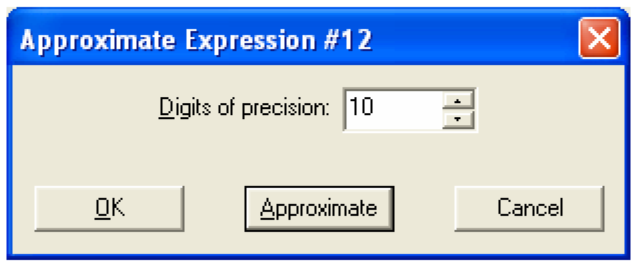

Figure 3: "Approximate" Dialog Box

Try This:

Highlight $\theta = \dfrac{\pi}{6}$ and approximate it to 15 decimal places. You should see $\theta = 0.523598775598298$ as your result.

Parentheses:
Round parentheses ( ) and square brackets [ ] are **NOT** equivalent in Derive. Use only parentheses for grouping expressions. You may need to use more than one set of parentheses for some expressions.

Try This:

Author the expression $\dfrac{(2x+6)^2 - 8x}{3x+2}$. Then choose "Basic" from the Simplify menu. You should see $\dfrac{4(x^2+4x+9)}{3x+2}$ as your result. What happens if you try to get Derive to distribute the 4 by choosing "Expand" from the Simplify menu?

Author the expression $\left[3-(6+2x)^2\right]^2 - (2x-1)^4$, and simplify it. Remember to use only round parentheses (not square brackets) in the expression entry line!! You should see $224x^3 + 816x^2 + 1592x + 1088$ as your result.

Author the expression $3-(-5)$ and simplify it. The result should be 8. Now author the expression $3-[-5]$ (with square brackets) and simplify it. The result is $[5]+3$.

Radical Expressions:
You can access the square root symbol (√) from the math symbol toolbar, or by typing "sqrt" in the expression entry line. Be sure to use parentheses appropriately. Typing "`√x+3`" will result in "$\sqrt{x}+3$." To get "$\sqrt{x+3}$," you must type "`√(x+3)`". For other roots (like cube roots), you must use exponential notation: type "`x^(1/3)`" for "$\sqrt[3]{x}$," for example.

Try This:
Author the equation $\sqrt{x^2+4}+6=2x-1$, and then solve it. You should get $x=\frac{\sqrt{61}}{3}+\frac{14}{3}$ as your result.

Author the equation $\sqrt{2x-\sqrt[3]{x+1}}=3$, and then solve it. You should see $x=\left(\frac{11}{32}-\frac{\sqrt{9795}}{288}\right)^{1/3}+\left(\frac{\sqrt{9795}}{288}+\frac{11}{32}\right)^{1/3}+\frac{9}{2}$ as your result. Be sure to read the section below on "Copying Results from Derive" for the proper way to record this result on your homework paper.

Roots of Negative Numbers:
In the real number system, $\sqrt[3]{-8}=-2$, because $(-2)^3=-8$. In the complex number system, there are two other numbers whose cube is -8, and it is one of these that Derive returns when we ask it to simplify $(-8)^{1/3}$. (Try it!) We won't be using Derive to compute any roots of negative numbers – this is an instance in which we have to be smarter than the computer. For example, if we want to use $\sqrt[3]{x^2}$, we should type "`(x^2)^(1/3)`" in the expression entry line, and not "`x^(2/3)`." (Note that even if x is negative, x^2 is not.) If n is an odd number, we can use the fact that $\sqrt[n]{-x}=-\sqrt[n]{x}$ to compute the n^{th} root of a particular negative number.

Try This:
Author and simplify the expressions $(-216)^{2/3}$ and $\left((-216)^2\right)^{1/3}$. The result of simplifying the first expression should be $-18+18i\sqrt{3}$, and the result of simplifying the second should be 36.

Find the value of $\sqrt[5]{-2861381721051424}$ by authoring and simplifying the expression $-(2861381721051424)^{1/5}$. The result should be -1234.

Absolute Value:
The command for the absolute value function is "abs." So to get $|x-3|$, type "`abs(x-3)`" in the expression entry line.

Defining Functions:
If you have a function that you'll be using more than once, you may choose to save it in Derive. From the Author menu, select "Function Definition." Type the name of the function, like "$f(x)$" or "$y(t)$" in the "Function Name and Argument" line. Type the rest of the function, like "x^2+3," in the "Function Definition" space. (See Figure 4.) Then click "OK." You should see "$f(x):=x^2+3$" in the algebra window. You can now use this function when authoring expressions.

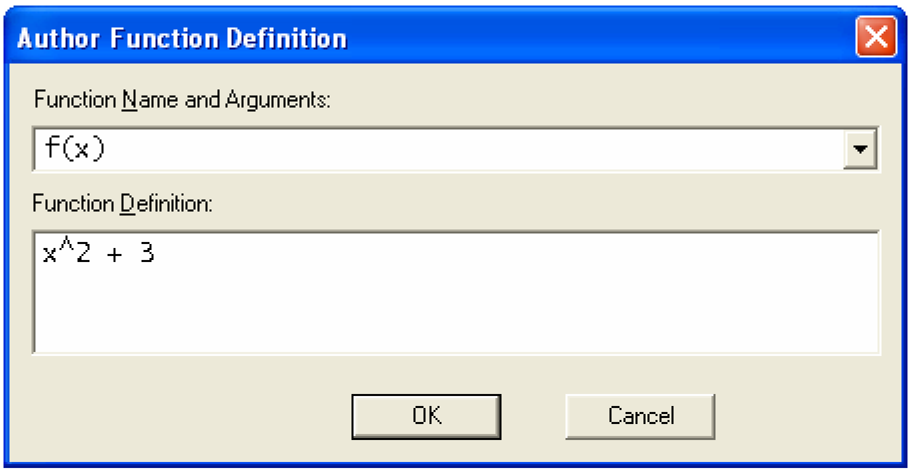

Figure 4: "Author Function" Dialog Box

Try This:
Define the function $f(x)=x^2+3$ as described above. Then author the expression $f(2)$. Choose either "Expand" or "Basic" from the Simplify menu to get the result of 7.

Now author $f(a+h)$ and simplify. You should see $a^2+2ah+h^2+3$ as your result.

Author the expression $g(x)$. When you press "Enter," you should see $g \cdot x$ in the algebra window. Derive interprets both *g* and *x* as variables unless you first define *g* as a function.

Copying Results from Derive onto Your Homework Paper:
The square root symbol in Derive is "√", and is frequently used in conjunction with parentheses. You should use conventional root notation when copying results onto your paper. That is, write "$\sqrt{x+3}$," instead of "$\sqrt{}(x+3)$." Recall that Derive doesn't have any built-in symbols for third, fourth, or higher roots. Your teacher will tell you whether to copy "$x^{2/3}$" as "$\sqrt[3]{x^2}$," or to leave it as is.

Even though Derive doesn't use square brackets for grouping, you should feel free to use them on your paper if it makes your expression more readable. For example, you may write $\left[(2+x)^4+6\right]^8$ instead of $\left((2+x)^4+6\right)^8$.

Exercises

Use *Derive*[TM] *6* for the following exercises. Use standard mathematical notation to record the results ***on a separate sheet of notebook paper.*** Do not turn in a print-out of your Derive session. Note that "standard" notation is not always the same as Derive's notation. Remember that I should be able to tell what the question was, from the answer written on your paper.

1. Simplify each of the following. If Derive returns a non-real complex number, you'll need to do something in order to get a real number answer.

(a) $\sqrt[4]{\dfrac{14641}{2401}}$ (b) $\sqrt[3]{-3375}$

2. Define the functions $f(x) = 2\sqrt{3x-1}$ and $g(x) = x^3$. Simplify each of the following.

(a) $f(14)$ (b) $f(g(x))$ (c) $g(f(x))$ (d) $g(a+2)$

(e) $g(a)+2$ (f) $g(a+2)-g(a)$

3. What do the results of Questions 2(d) and 2(e) tell you?

4. Approximate $\sqrt[5]{14^2}$ with 14 digits of precision.

5. How many times does the number 4 appear in the first 100 digits of π?

6. Simplify the expression $\left(3x + \sqrt{|x+2|-4}\right)^2$. Use a "Basic" simplification – you won't like the result you get if you use "Expand."