The purpose of these exercises is to compute the derivatives of composite functions: those functions that have the form $y = f\left(g(x)\right)$, where $f(x)$ and $g(x)$ are differentiable functions.

Exponential Functions in Derive:
Exponential functions like 2^x don't require any special treatment in Derive. The constant, *e*, on the other hand takes a little extra effort**. If you type the letter *e* from the keyboard, you'll be authoring a variable, and not a constant.** Get the constant *e* from the math symbol palette at the bottom of the algebra window. If you aren't sure which "e" you've authored, look closely in the algebra window. The constant is italicized, but the variable is not. You can also author the function e^x by typing "`exp(x)`" in the expression entry line. ("Exp" stands for *exponential*.) The parentheses are a good idea, but are not strictly necessary.

Logarithmic Functions in Derive:
Author the natural logarithm function by typing "`ln(x)`" in the expression entry line. The parentheses are a good idea, but are not strictly necessary. To author logarithms with bases other than *e*, type "`log(x,a)`" in the expression entry line, where *a* is the base of the logarithm. In this case, the parentheses are necessary.

Try This:
Author the expression $\log_3 x$. You should see $\text{LOG}(x,3)$ as your result. If you simplify the result, you'll see something with natural logarithms. So don't simplify this.

Author the expression $3e^{4x}$ by typing "`3exp(4x)`" in the expression entry line. Simplify the result to see $3e^{4x}$.

Copying Results from Derive onto Your Homework Paper:
As we've seen before, Derive tends to use upper case letters for functions that mathematicians usually express using lower case letters. Where Derive writes $\text{EXP}(x)$, you should write e^x. Derive shows $\text{LN}(x)$, but you should write $\ln x$ or $\ln(x)$. And you should definitely convert Derive's $\text{LOG}(x,3)$ to the standard $\log_3 x$. You also need to beware of Derive's use of parentheses. Write $(\ln x)^2$ when you see $\text{LN}(x)^2$ in Derive. This is NOT the same as $\ln x^2 = \ln(x^2)$.

Exercises:
Use *Derive*^{TM} *6* for the following exercises. Use standard mathematical notation to record the results ***on a separate sheet of notebook paper.*** Do not turn in a print-out of your Derive session. Pay attention to the manner in which you record your results. For example, you should ***not*** indicate that a function and its derivative are equal.

1. Find the derivatives of the following functions. Don't author the "$f(x) =$" or "$g(x) =$" part of the function.

(a) $f(x) = \ln x$ (b) $f(x) = \log_6 x$ (c) $f(x) = 2^x$ (d) $f(x) = \sqrt{x}$

(e) $g(x) = x^4 + 3$ (f) $g(x) = x^3 + 1$ (g) $g(x) = \dfrac{1}{x}$ (h) $g(x) = e^x$

2. For each pair of functions *f* and *g* shown below, write the composite function $y = f\left(g(x)\right)$. Then author the expression for *y* and use Derive to find $\dfrac{dy}{dx}$.

(a) $f(x) = \ln x$, $g(x) = x^4 + 3$ (b) $f(x) = \log_6 x$, $g(x) = x^3 + 1$

(c) $f(x) = 2^x$, $g(x) = \dfrac{1}{x}$ (d) $f(x) = x^4 + 3$, $g(x) = \ln x$

3. Complete the following sentence: Based on the results of Questions 1 and 2, I believe that if $y = f\left(g(x)\right)$, then $\dfrac{dy}{dx} =$ _____. Your formula should not include symbols other than $f(x)$, $g(x)$, $f'(x)$, $g'(x)$, $f\left(g(x)\right)$, $f'\left(g(x)\right)$, $+$, $-$, and $\cdot$.

4. Differentiate each of the following functions. Identify the component functions *f* and *g*, and determine whether your formula from Question 3 works for these derivatives. All of the f and g functions you need are in Question #1.

(a) $y = \sqrt{x^3 + 1}$ (b) $y = \dfrac{1}{\ln x}$ (c) $y = e^{x^4+3}$

5. Differentiate each of the following functions. Identify the component functions *f* and *g*, and determine whether your formula from Question 3 works for these derivatives. Derive will perform some simplification, so the only way to know if your formula works is to see if you can get from your formula to Derive's result. (Show your work!) All of the f and g functions you need are in Question #1.

(a) $y = \sqrt{x^4 + 3}$ (b) $\ln\left(\sqrt{x}\right)$