MT 135-56/8 **Exploring Functions with *Derive*^{TM} *6*** **Fall, 2004**

The purpose of these exercises is to use Calculus and Derive to explore properties of various functions.

Copying and Pasting in Derive:

If you haven't already discovered this, you should know that you can use the right-click button on the mouse to copy highlighted expressions from the algebra window and paste them into the expression entry line. This feature will be a big time saver when you do the exercises below.

Solving Inequalities in Derive:

The procedure for solving inequalities in Derive is like that for solving equations. The only difference is that the expression will include one of the symbols $>, <, \le$, or $\ge$ instead of an equal sign. All of these symbols are available from the math symbol toolbar at the bottom of the algebra window. You can also access them from the keyboard: Type "<=" for "$\le$" and ">=" for "$\ge$."

Try This:

Author the expression $\dfrac{x^2-3x+2}{x+4} \ge 0$. Then solve it over the real numbers. You should see the result $-4 < x \le 1 \vee x \ge 2$. Recall that the symbol "$\vee$" means "or."

Higher Order Derivatives in Derive:

You can compute the second, third, fourth, etc. derivative of a function in Derive by changing the "order" in the "Differentiate" dialog box from 1 to 2, 3, 4, etc.

Try This:

Author the expression $\ln(x^2+1)$. Compute $\dfrac{d^4}{dx^4}\left(\ln(x^2+1)\right)$. After you press "OK," you should see $\left(\dfrac{d}{dx}\right)^4 \mathrm{LN}(x^2+1)$ in the algebra window. When you simplify, you should get the result $-\dfrac{12(x^4-6x^2+1)}{(x^2+1)^4}$.

Copying Results from Derive onto Your Homework Paper:
Derive gives the solutions to inequalities in terms of inequalities. But you know that the solution set for an inequality is really a *set*, and so you should record the result as a set, either in set notation, or (preferably) in interval form. So you should write the solution to $\frac{x^2-3x+2}{x+4} \geq 0$ as $\left(-4,1\right] \cup \left[2,\infty\right)$. The "union" symbol between the two intervals may be inappropriate, depending on your reason for solving the inequality. For example, it's possible that $f'(x) \geq 0$ on $\left(-4,1\right] \cup \left[2,\infty\right)$, but this means that f is increasing each of the intervals $\left(-4,1\right]$ and $\left[2,\infty\right)$, and not on $\left(-4,1\right] \cup \left[2,\infty\right)$.

Regarding derivative notation, you should write $\frac{d^2}{dx^2}(x^3-8x)$, and not $\left(\frac{d}{dx}\right)^2(x^3-8x)$.

Exercises:
Use *Derive*^{TM} *6* for the following exercises. Use standard mathematical notation to record the results ***on a separate sheet of notebook paper.*** Do not turn in a print-out of your Derive session. The work that you turn in should show the steps and thought processes that lead to your answers.

1. Determine the interval(s) on which each of the following functions is increasing.

   (a) $f(x) = 3x^4 - 22x^3 + 3x^2 + 12x - 15$

   (b) $g(x) = 2x^3 - 6x - e^{2x-1}\left(x^2 - x - \frac{1}{2}\right)$

2. Determine the interval(s) on which each of the following functions is concave down.

   (a) $f(t) = \frac{t^3}{6} + 6t^2 + \left(t^3 - 12t^2\right)\ln t$

   (b) $g(t) = \frac{1}{6}t^3 - 3t^2 - 7t + (7t+14)\ln(t+2)$

3. Find all relative extrema, inflection points, and horizontal asymptotes of the function $f(x) = \frac{3x+2}{\sqrt{x^2+1}}$.