MT 135-56/8 **Derivatives with *Derive*™ *6* , Part II** **Fall, 2004**

The purpose of these exercises is to compute the derivatives of functions of the form $f(x)=u(x)\cdot v(x)$, where $u(x)$ and $v(x)$ are differentiable functions. Although we won't formally discuss trigonometric functions in MT 135, we will use some of them in the exercises below because they are good for demonstration purposes.

Trigonometric Functions in Derive:
All six trigonometric functions (sine, cosine, tangent, secant, cosecant, and cotangent) are preexisting functions in Derive. Author them by using their standard abbreviations in the expression entry line. For example, type "`sinx`" to author $\sin x$.

Derivatives in Derive:
To differentiate a highlighted expression, choose "Differentiate" from the Calculus menu. In the "Differentiate" dialog box (see Figure 1), choose the variable of differentiation from the drop-down menu, make sure "Order" is set at 1, and click "OK." Then do a "Basic" simplification from the Simplify menu.

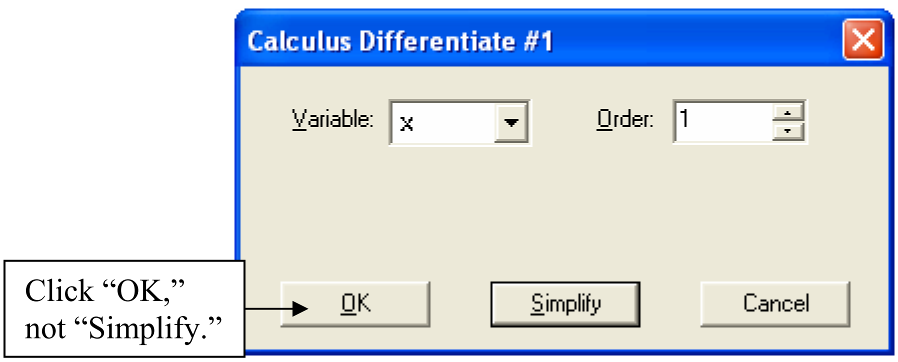

Figure 1: "Differentiate" Dialog Box

Try This:
Suppose $u(x)=\sin x$. To find $u'(x)$, author the expression $\sin x$ (without the "$u(x)$"). Then differentiate it as above. After you click "OK," you should see the result $\frac{d}{dx}\sin(x)$. Verify that this is correct, and then simplify to get the result final result $\cos(x)$.

Copying Results from Derive onto Your Homework Paper:
Derive uses upper case letters for trig functions. Standard mathematical notation uses lower case letters. Write "$\tan(x)$" instead of "$\mathrm{TAN}(x)$." Also, you need not use parentheses if the argument is a single term: "$\tan x$" is appropriate. Beware of Derive's use of parentheses in other instances, though. When Derive shows "$\mathrm{COS}(x)^2$," it means "$(\cos x)^2$," and not "$\cos x^2 = \cos(x^2)$." You should write "$(\cos x)^2$" on your paper in this case.

Exercises:
Use *Derive*^{TM} *6* for the following exercises. Use standard mathematical notation to record the results ***on a separate sheet of notebook paper.*** Do not turn in a print-out of your Derive session. For each problem, your solutions should indicate both the function and its derivative. Pay attention to the manner in which you record your results. For example, you should ***not*** indicate that a function and its derivative are equal.

1. Find the derivatives of the following functions:

(a) $u(x) = x^2$ (b) $u(x) = x^3$ (c) $u(x) = x^4$ (d) $u(x) = x^{-1}$
(e) $v(x) = \sin x$ (f) $v(x) = \cos x$ (g) $v(x) = \tan x$ (h) $v(x) = \cot x$

2. Differentiate each of the following functions. Notice that each of these is the product of two of the functions from Question 1. After you differentiate, identify functions $u(x)$ and $v(x)$ in the expression for $f(x)$. Then identify functions $u(x)$, $v(x)$, $u'(x)$, and $v'(x)$ in the expression for $f'(x)$.

(a) $f(x) = x^2 \sin x$ (b) $f(x) = x^3 \cos x$ (c) $f(x) = x^4 \tan x$

3. Complete the following sentence: Based on the results of Questions 1 and 2, I believe that if $f(x) = u(x) \cdot v(x)$, then $f'(x) =$ ________________________________ . Your formula should not include symbols other than $u(x)$, $v(x)$, $u'(x)$, $v'(x)$, +, −, and $\cdot$. (You might not use all of these symbols.)

4. Use your formula from Question 3 to differentiate each of the following functions. Then have Derive differentiate the function. Determine whether your formula from Question 3 works for these derivatives. Derive may perform some simplification, so analyze your results carefully.

(a) $f(x) = x^{-1} \cot x$ (b) $f(x) = x^4 \sin x$

5. Differentiate each of the following functions. Determine whether your formula from Question 3 works for these derivatives. Derive will definitely perform some simplification, so the only way to know if your formula works is to see if you can get from your formula to Derive's result. (Show your work!)

(a) $f(x) = x^2 \cdot x^4$ (b) $f(x) = x^3 \cdot x^{-1}$