THE EASY WAY TO INTERACTIVE MATHEMATICS ON THE WEB

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Interactive mathematics activities allow students to experiment with mathematical concepts and form conjectures about general results. We use guided discovery activities in our calculus classes, for example, to lead students to a formulation of the product rule for derivatives. Such interactive activities can be implemented in a variety of settings: graphing calculators, computer algebra systems, and the internet, to name a few. Our focus here is using MathView software to create interactive web pages. We have chosen MathView for several reasons. The first is the immediacy with which students can see results of their experimentation. We also like MathView because students can access the pages from most internet browsers with the appropriate plug-in (which is available free of charge), and therefore are not restricted to using campus computer labs to do activities. Moreover, web pages generally allow students to see all the relevant information on one screen – a feature that cannot be duplicated on a graphing calculator. But perhaps our most compelling reason for using MathView is that writing MathView notebooks requires no knowledge of programming or use of special syntax.

Some of the activities we have used with our classes involve manipulation of graphs, derivative rules, and linear transformations. For example, in calculus or precalculus classes, we have students work with a web page that examines the graphs of functions like $f(x) = a(x-h)^2 + k$. The students change the values of *a*, *h*, and *k*, singly and then together, and observe and describe the results. Another web page allows students to examine the difference quotient and derivative of $f(x) = x^n$ for integer values of *n*. After experimenting with several values of *n*, most students are able to state the general power rule for derivatives. Linked to that page is another page that allows students to experiment with non-integer powers of *x*. Students can test their conjectured statement of the power rule as well as conclude that it holds only for constant powers. We have also developed pages that allow students to discover the product, quotient, and chain rules for derivatives, as well as some pages for linear algebra classes in which students observe the effects of linear transformations on geometric figures in the plane. When we want students to discover some mathematical property, we usually provide the students with worksheets that lead them through the initial stages of experimentation. Samples of the worksheets and web pages are available on the world wide web at `http://www.jcu.edu/math/ictcm99/`.

For those who are interested in designing their own activities using MathView, we provide some step-by-step instructions below. The notebooks discussed here were written using MathView, version 2.5. We begin with an activity involving the product rule. The mathematical statements in MathView are called *propositions*. In a new MathView notebook, we create a proposition for the function $f(x) = x^2$. After **clarifying** the notebook (from the **Notebook** menu) so that MathView recognizes $f(x)$ as a function, we create a proposition for $\frac{d}{dx} f(x)$ by using the derivative button on the MathView palette. We substitute the expression for $f(x)$ into the derivative proposition by clicking once on the equal sign, and then holding the control key while dragging the function definition over the derivative statement. When we release the mouse button, we see the simplified derivative of $f(x)$. We repeat this process for $g(x) = \sin(x)$. We then create $h(x) = f(x) \cdot g(x)$, and substitute the expressions for $f(x)$ and $g(x)$ into $h(x)$. Finally, we make a proposition for $\frac{d}{dx} h(x)$, and substitute the expression for $h(x)$ into this. The result appears in Figure 1 below.

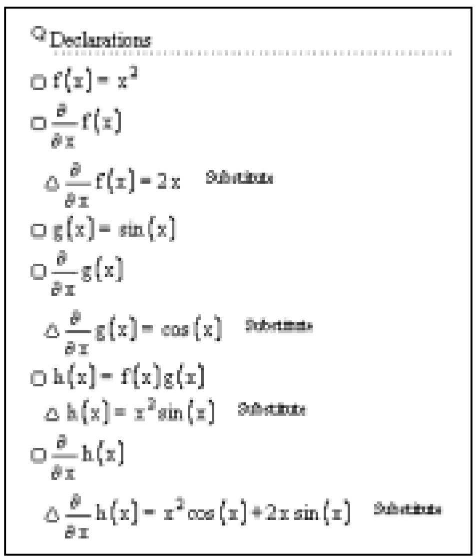

Figure 1: Product Rule Notebook

We can then make changes to either $f(x)$ or $g(x)$, and observe the changes in $\frac{d}{dx} h(x)$. (See Figures 2 and 3.) In the guided discovery activities that accompany this page, we ask the students to make changes to both $f(x)$ and $g(x)$, and then to write an expression for $\frac{d}{dx} h(x)$ using only the symbols $f(x)$, $g(x)$, $f'(x)$, and $g'(x)$. We must choose our functions $f(x)$ and $g(x)$ carefully, so that MathView will not simplify the result in such a way that students will be unable to recognize the component functions in the derivative of *h*.

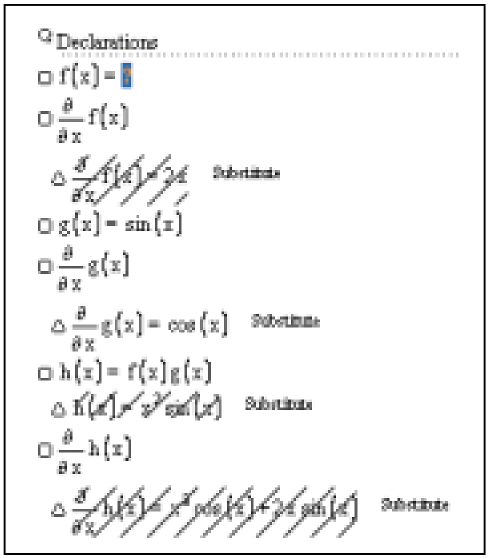

Figure 2: A change in progress

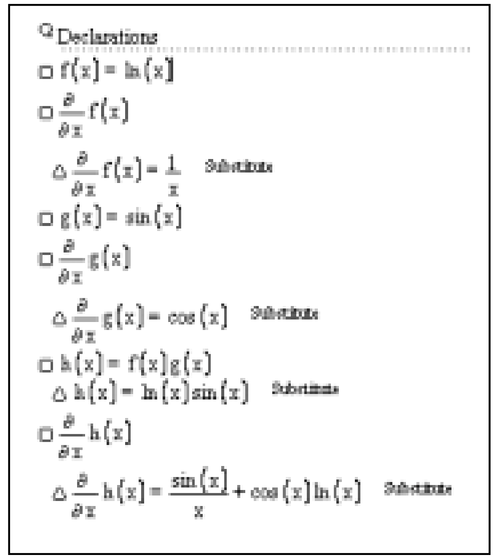

Figure 3: After the change

For those teaching linear algebra, we include directions creating a for a MathView notebook for visualizing linear transformations in the plane. In a new notebook, we define a matrix, called "*square*" here, containing the vertices of a square. The vertices are represented as column vectors. MathView will draw the square by joining the points in order, so the first vertex must appear twice – in the first column and again in the last. In MathView, we enter a matrix by typing the elements of the first row, separated by commas, followed by a semicolon and the elements of the second row, again separated by commas. We also define a matrix, *A*, that represents a linear transformation. After clarifying the notebook to make *A* and *square* user-defined variables, we define a new matrix, *B*, as the product of *A* and *square*. We clarify again, and are ready to draw the polygon.

MathView will not allow us to open a graphing window directly, so we type the dummy equation $y = x$, highlight it, and choose "**y=f(x) ... linear**" from the **Graph** menu. Since we'll need four line segments to complete our square, we choose "**Additional ... Add Line Plot**" from the **Graph** menu three times. We can now delete the $y = x$ equation. We open the graph details (the icon on the graph that looks like a paragraph), and change each line so that it draws one side of the square. (See Figure 4 below.) We also require that the graph show the true proportions, so that our square looks square. After a final clarification of the notebook, we close the graph details and are ready to experiment with different values in the matrix *A*. As we change the linear transformation A, the graph shows the effect of these changes on our original square. (See Figures 5 and 6.)

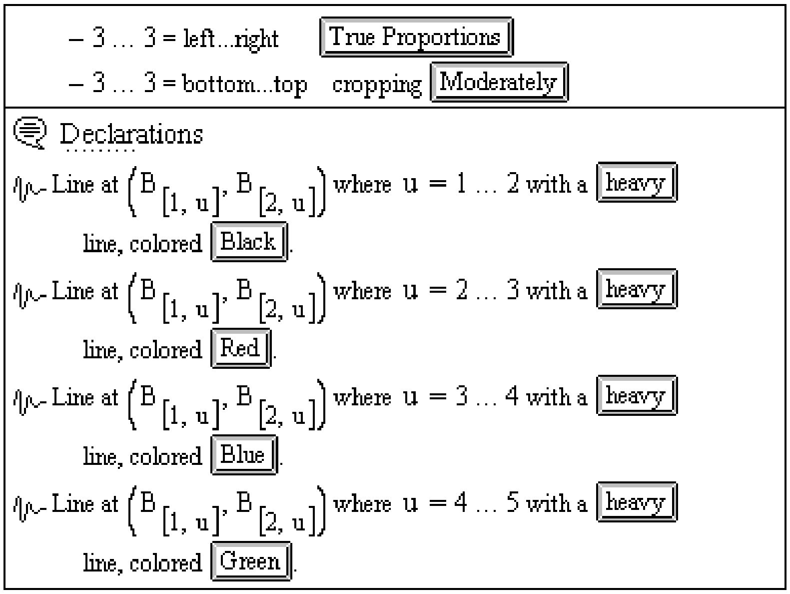

Figure 4: Graph details

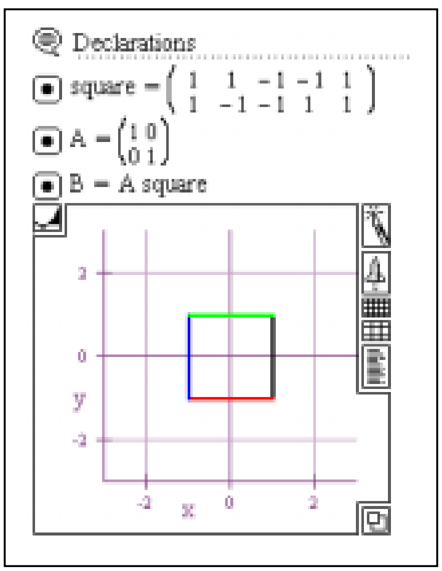

Figure 5: Original square

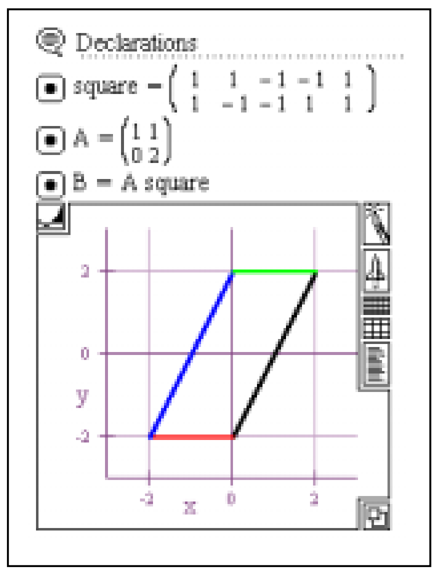

Figure 6: After a change

After we have a MathView notebook in the form we would like to use on the Web, we save the notebook as a .THP file. We create a web page using our favorite software. To include the interactive portion, we open our notebook in MathView, and copy the HTML tag from the **Edit** menu. We insert this tag in our web page – "insert HTML tag" is an option in our web page editor, or we could insert the tag directly into the HTML source code.

Finally, we tweak the web page in a few places by editing the HTML code directly. Where we have inserted the tag to the MathView notebook, the HTML code will contain a link to the “pluginspage.” Any user who does not have the proper plug-in will be led to this link when he or she tries to load our page. Since the default link supplied by MathView, version 2.5, is no longer valid, we change it to point to a page containing a copy of the plug-in. The other change we make to the HTML code has to do with the size of the inserted image. Since the MathView portion of a web page is not static, the size allotted to the original image may not be sufficient to contain all the mathematics resulting from a change made by the user. We increase the height and width of the embedded image to allow for these changes. (See Figure 7.) When the pages meet with our approval, we put the HTML document containing the web page, along with the MathView notebook(s) to which it is linked, in a single directory on our university’s webserver.

```
<p><embed src="powrule.thp" height=450 width=900
pluginspage="http://www.livemath.com/">
```

Figure 7: HTML code

At the time this paper was written in November, 1999, MathView had been acquired by Web Primitives, LCC. The software is being upgraded and should be released soon, under the product name LiveMath Maker. The plug-in needed to view the web pages is currently available from the LiveMath website: `http://www.livemath.com/`.