

## Sample student worksheet on discovering the product rule.

In this worksheet you will use an interactive web page to help you deduce a formula for the derivative of a product of functions,  $f(x)g(x)$ . Using a PC connected to the Internet, point your web browser (Netscape or Internet Explorer) to

<http://www.jcu.edu/cspitzna/calculus/>

From there, click on the link to *The Product Rule*. Use this web page perform the experiments below, and to help you answer the corresponding questions. ***Be sure to read the questions and directions carefully.***

1. On the web page you will see a function  $f(x)$  and its derivative,  $\frac{\partial}{\partial x} f(x)$ . You may not be familiar with these functions, but that will not impair your ability to continue with these questions!

What is the function  $f(x)$  presented on the web page?

What is its derivative?

There is also a function  $g(x)$  and its derivative,  $\frac{\partial}{\partial x} g(x)$ .

What is the function  $g(x)$ ?

What is its derivative?

2. The web page also shows the product function,  $f(x)g(x)$ . What is this function?

What is the derivative of the product,  $\frac{\partial}{\partial x} f(x)g(x)$ ?

3. Look carefully at the formula for  $\frac{\partial}{\partial x} f(x)g(x)$  and the expressions for the derivatives of  $f$  and  $g$ . Try to recognize the original functions and their derivatives as parts of this formula. Write an expression for  $\frac{\partial}{\partial x} f(x)g(x)$  that uses only the symbols  $f(x)$ ,  $g(x)$ ,  $\frac{\partial}{\partial x} f(x)$  and  $\frac{\partial}{\partial x} g(x)$ .

- Based on your observation in question 3, formulate a conjecture about the derivative of a product function  $f(x)g(x)$ .
- Now change the definition of  $f(x)$  on the web page. To do this, click on the square box in front of the line that defines  $f$ , and then press the right arrow cursor key. (This should move the cursor to the end of that line.) Then press the left arrow cursor key until you can change the “sin” in the definition of  $f$  to “cos”. The mathematics below should change instantly. (If it appears to get hopelessly messed up, just click RELOAD or REFRESH on your web browser, and repeat this step.)

Fill in the formulas for the following:

$$f(x) \qquad \frac{\partial}{\partial x} f(x)$$

$$g(x) \qquad \frac{\partial}{\partial x} g(x)$$

$$f(x)g(x) \qquad \frac{\partial}{\partial x} f(x)g(x)$$

- Summarize your findings about  $\frac{\partial}{\partial x} f(x)g(x)$ . Does your conjecture from question 4 still hold?

- Now change the formula for  $g(x)$  to “ln(x)”. To do this, click on the box in front of the definition of  $g(x)$ , and then press the right arrow cursor key. Press the backspace key twice, so that the power is deleted and the  $x$  is highlighted. Then type LN(X) (using lower case letters).

Fill in the formulas for the following:

$$f(x) \qquad \frac{\partial}{\partial x} f(x)$$

$$g(x) \qquad \frac{\partial}{\partial x} g(x)$$

$$f(x)g(x) \qquad \frac{\partial}{\partial x} f(x)g(x)$$

8. Summarize your findings about  $\frac{\partial}{\partial x} f(x)g(x)$ . Is the derivative of  $f(x)g(x)$  found by the same formula as in questions 4 and 6?
9. Suppose someone tells you that if  $f(x) = 3^x$ , then  $f'(x) = 3^x \ln(3)$ . (You don't even have to understand this formula!!) Use your conjecture about the derivative of a product to find an expression for the derivative of the function  $F(x) = x^4 \cdot 3^x$ .