

# Belief Logic

$u:A$	=	You believe that A.
	=	You accept A.
$\underline{u}:A$	=	Believe that A.
	=	Accept A.

1. The result of writing a small letter and then “:” and then a wff is a *descriptive* wff.
2. The result of writing an underlined small letter and then “:” and then a wff is an *imperative* wff.

You believe that A is true =  $u:A$

You don't believe that A is true =  $\sim u:A$

You believe that A is false =  $u:\sim A$

You don't believe A and you don't believe not-A =  $(\sim u:A \cdot \sim u:\sim A)$

You believe that you ought to do A =  $u:OAA$

Everyone believes that they ought to do A =  $(x)x:OAx$

You believe that if A then not-B =  $u:(A \supset \sim B)$

If you believe A, then you don't believe B =  $(u:A \supset \sim u:B)$

$$\begin{aligned}
\text{Believe that A is true} &= \underline{u}:A \\
\text{Don't believe that A is true} &= \sim\underline{u}:A \\
\text{Believe that A is false} &= \underline{u}:\sim A \\
\text{Don't believe A and don't believe not-A} &= (\sim\underline{u}:A \cdot \sim\underline{u}:\sim A)
\end{aligned}$$

$$\begin{aligned}
\text{Believe that you ought to do A} &= \underline{u}:OA\underline{u} \\
\text{Let everyone believe that they ought to do A} &= (\underline{x})\underline{x}:OA\underline{x}
\end{aligned}$$

$$\begin{aligned}
\text{If you in fact believe A, then don't believe B} &= (\underline{u}:A \supset \sim\underline{u}:B) \\
\text{Don't combine believing A with believing B} &= \sim(\underline{u}:A \cdot \underline{u}:B)
\end{aligned}$$

# Three Approaches to Belief Logic

1. Belief logic studies what belief formulas validly follow from what other belief formulas.
2. Belief logic studies how people would believe if they were *completely consistent believers*.
3. Belief logic generates *consistency imperatives*, like:
  - “Don’t combine believing A with believing not-A”  
“ $\sim(\underline{u}:A \cdot \underline{u}:\sim A)$ ”
  - “Don’t combine believing A-and-B with not believing A”  
“ $\sim(\underline{u}:(A \cdot B) \cdot \sim\underline{u}: A)$ ”

*Belief worlds* are represented by strings of one or more instances of a small letter – for example, “u,” “uu,” “uuu,” and so on.

A *belief policy* is a set of imperatives about what someone (typically a generic “you”) is or is not to believe (for example, “Believe that Michigan will play; be neutral about whether Michigan will win”).

A belief policy is *consistent* if and only if (1) the set S of things that the person is told to believe is logically consistent, and (2) the person isn’t forbidden to believe something that follows logically from set S.

A *belief world* (relative to a belief policy) is a possible world that contains everything the person is told to believe.

“Don’t combine believing A with believing not-A.”

	$[\therefore \sim(\underline{u}:A \cdot \underline{u}:\sim A)$	<b>Valid</b>
*	1	asm: $(\underline{u}:A \cdot \underline{u}:\sim A)$
	2	$\therefore \underline{u}:A$ {from 1}
	3	$\therefore \underline{u}:\sim A$ {from 1}
	4	$u \therefore A$ {from 2} $\Leftarrow$
	5	$u \therefore \sim A$ {from 3} $\Leftarrow$
	6	$\therefore \sim(\underline{u}:A \cdot \underline{u}:\sim A)$ {from 1; 4 contradicts 5}

B+ If you’re told to believe A, then put A in all of your belief worlds.

“Don’t combine believing A-and-B with not believing A.”

		$[\therefore \sim(\underline{u}:(A \cdot B) \cdot \sim\underline{u}:A)$	<b>Valid</b>
*	1	$\text{asm: } (\underline{u}:(A \cdot B) \cdot \sim\underline{u}:A)$	
	2	$\therefore \underline{u}:(A \cdot B) \quad \{\text{from 1}\}$	
*	3	$\therefore \sim\underline{u}:A \quad \{\text{from 1}\}$	
	4	$u \therefore \sim A \quad \{\text{from 3}\} \quad \Leftarrow$	
	5	$u \therefore (A \cdot B) \quad \{\text{from 2}\}$	
	6	$u \therefore A \quad \{\text{from 5}\} \quad \Leftarrow$	
	7	$\therefore \sim(\underline{u}:(A \cdot B) \cdot \sim\underline{u}:A) \quad \{\text{from 1; 4 contradicts 6}\}$	

B- If you’re told to *refrain* from believing A, then put not-A in a *new* belief world of yours.

# Belief Inference Rules

B-

$\sim \underline{u}:A \rightarrow u \therefore \sim A,$   
use a *new* string of u's

\*

First drop negative imperative belief operators; use a new belief world each time.

B+

$\underline{u}:A \rightarrow u \therefore A,$   
use any string of u's

Then drop positive imperative belief operators; use old belief worlds if you have them (otherwise use a new world “u”).



$u:A$  = You accept (endorse, assent to, say  
in your heart) “A is true.”  
= You believe that A.  
  
 $u:\underline{A}$  = You accept (endorse, assent to, say  
in your heart) “Let act A be done.”  
= You will that act A be done.

*If A is present:*  $u:A\underline{u}$  = You accept the imperative for you to do A now.  
= You act (in order) to do A.

*If A is future:*  $u:A\underline{u}$  = You accept the imperative for you to do A in  
the future.  
= You’re resolved to do A.

*If  $u \neq x$ :*  $u:A\underline{x}$  = You accept the imperative for X to do A.  
= You desire (or want) that X do A.

$u:Au$  = You act (in order) to do A.  
 $u:A\underline{u}$  = You say in your heart, “Do A” (addressed to yourself).

$Au$  = You do A.

$u:(\exists x)(Kx \cdot R\underline{x})$  = You desire that some who kill *repent*.  
 = You say in your heart “Would that some who kill *repent*.”

$u:(\exists x)(K\underline{x} \cdot Rx)$  = You desire that some *kill* who repent.  
 = You say in your heart “Would that some *kill* who repent.”

$u:(\exists x)(K\underline{x} \cdot R\underline{x})$  = You desire that some both *kill* and *repent*.  
 = You say in your heart “Would that some *kill* and *repent*.”

$\underline{u}:\underline{A}$  = Accept (endorse, assent to, say in your heart) “Let act A be done.”  
 $\underline{u}:\underline{A}$  = Will that act A be done.

*If A is present:*  $\underline{u}:\underline{A}\underline{u}$  = Accept the imperative for you to do A now.  
 $\underline{u}:\underline{A}\underline{u}$  = Act (in order) to do A.

*If A is future:*  $\underline{u}:\underline{A}\underline{u}$  = Accept the imperative for you to do A in the future.  
 $\underline{u}:\underline{A}\underline{u}$  = Be resolved to do A.

*If  $u \neq x$ :*  $\underline{u}:\underline{A}\underline{x}$  = Accept the imperative for X to do A.  
 $\underline{u}:\underline{A}\underline{x}$  = Desire (or want) that X do A.

Use underlining *before* “:” to *tell* someone what to believe or will.

Use underlining *after* “:” if the sentence is about *willing*.

*Indicatives*

u:A = You believe A.

u:A = You will A.

*Imperatives*

u:A = Believe A.

u:A = Will A.

Don't combine *believing* that it's wrong  
for you to do A with *acting* to do A.

	$[\therefore \sim(\underline{u}:O\sim A\underline{u} \cdot \underline{u}:A\underline{u})$	Valid
*	1	asm: $(\underline{u}:O\sim A\underline{u} \cdot \underline{u}:A\underline{u})$
	2	$\therefore \underline{u}:O\sim A\underline{u}$ {from 1}
	3	$\therefore \underline{u}:A\underline{u}$ {from 1}
	4	$u \therefore O\sim A\underline{u}$ {from 2}
	5	$u \therefore A\underline{u}$ {from 3}
	6	$u \therefore \sim A\underline{u}$ {from 4}
	7	$\therefore \sim(\underline{u}:O\sim A\underline{u} \cdot \underline{u}:A\underline{u})$ {from 1; 5 contradicts 6}

- = A is evident to you.
- Ou:A** = It's obligatory (rationally required) that you believe A.
- = Insofar as intellectual considerations are concerned (including your experiences), you ought to believe A.
  
- = A is reasonable for you to believe.
- Ru:A** = It's all right (rationally permissible) that you believe A.
- = Insofar as intellectual considerations are concerned (including your experiences), it would be all right for you to believe A.

It would be unreasonable for you to believe A =  $\sim R_{\underline{u}}:A$

It's obligatory that you not believe A =  $O_{\sim\underline{u}}:A$

It would be reasonable for you to take no position on A =  $R(\sim\underline{u}:A \cdot \sim\underline{u}:\sim A)$

It's evident to you that if A then B =  $O_{\underline{u}}:(A \supset B)$

If it's evident to you that A, then it's evident to you that B =  $(O_{\underline{u}}:A \supset O_{\underline{u}}:B)$

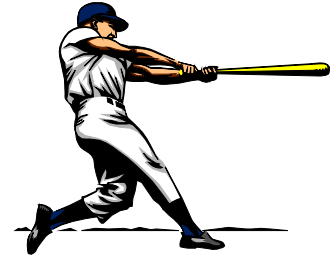
You ought not to combine believing A with believing not-A =  $O_{\sim(\underline{u}:A \cdot \underline{u}:\sim A)}$

*knowledge* = *evident true belief* [roughly]

You know that A = A is evident to you, A is true, & you believe A.

$uKA$  =  $(O_{\underline{u}}:A \cdot (A \cdot u:A))$

Hub = You hit the ball.  
Hub = Hit the ball.  
OHb = You ought to hit the ball.  
RHb = It's all right for you to hit the ball.



u:Hub = You believe that you'll hit the ball.  
u:Hub = Believe that you'll hit the ball.  
u:Hub = You act (with the intention) to hit the ball.  
u:Hub = Act (with the intention) to hit the ball.

Ou:Hub = You ought to believe (insofar as your evidence goes) that you'll hit the ball = It's evident to you that you'll hit the ball.  
Ru:Hub = It's all right (reasonable) for you to believe that you'll hit the ball (insofar as your evidence goes).

	1	$O_{\underline{u}}:G$	Valid	Theism is evident for you.
	[	$\therefore \sim R_{\underline{u}}:\sim G$		$\therefore$ Atheism is unreasonable
*	2	asm: $R_{\underline{u}}:\sim G$		for you.
	3	$D \therefore \underline{u}:\sim G$ {from 2}		
	4	$D \therefore \underline{u}:G$ {from 1}		
	5	$Du \therefore \sim G$ {from 3}	← using B+	
	6	$Du \therefore G$ {from 4}	← using B+	
	7	$\therefore \sim R_{\underline{u}}:\sim G$ {from 2; 5 contradicts 6}		

4. Reverse squiggles (quantificational/modal/deontic).  
5. Drop weak operators, using new things:  $\sim \underline{u}: R (\exists x) \diamond$   
6. Lastly, drop strong operators, using old things (if you have them):  $\underline{u}: O (x) \square$

[  $\therefore O \sim (\underline{u}: O \sim A \underline{u} \cdot \underline{u}: A \underline{u})$  Valid

- \* 1 asm:  $\sim O \sim (\underline{u}: O \sim A \underline{u} \cdot \underline{u}: A \underline{u})$
- \* 2  $\therefore R(\underline{u}: O \sim A \underline{u} \cdot \underline{u}: A \underline{u})$  {from 1}
- \* 3 D  $\therefore (\underline{u}: O \sim A \underline{u} \cdot \underline{u}: A \underline{u})$  {from 2}
- 4 D  $\therefore \underline{u}: O \sim A \underline{u}$  {from 3}
- 5 D  $\therefore \underline{u}: A \underline{u}$  {from 3}
- 6 Du  $\therefore O \sim A \underline{u}$  {from 4} ← using B+
- 7 Du  $\therefore A \underline{u}$  {from 5} ← using B+
- 8 Du  $\therefore \sim A \underline{u}$  {from 6}
- 9  $\therefore O \sim (\underline{u}: O \sim A \underline{u} \cdot \underline{u}: A \underline{u})$  {from 1; 7 contra 8}

You ought not to combine *believing* that it's wrong for you to do A with *acting* to do A.

1. Reverse squiggles (quantificational/modal/deontic).
2. Drop weak operators, using new things:  $\sim \underline{u}: R (\exists x) \diamond$
3. Lastly, drop strong operators, using old things (if you have them):  $\underline{u}: O (x) \square$

Our belief logic is oversimplified in three ways. A more sophisticated belief logic would:

- add qualifications to the implicit “One ought to be consistent” axiom and the derived consistency norms,
- perhaps qualify the conjunctivity principle (because of the lottery paradox), and
- add a second deontic operator  $O^*$  (for what one ought to believe insofar as intellectual considerations go) distinct from  $O$  (for what we ought to do all-things-considered).