The History of Logic

Aristotle (384–322 BC) invented logic.

- Predecessors: Fred Flintstone, geometry, sophists, pre-Socratic philosophers, Socrates & Plato.
- Syllogistic logic, laws of non-contradiction and excluded middle, modal logic.
- Some logic in a "wider sense": definitions, fallacies, inductive reasoning, etc.
- Parallels in "Buddhist logic."
- The basis for the "traditional logic" that dominated until the early 20th century.

The Stoics developed a rival logic.

- A propositional logic, focusing on “and,” “or,” and “if-then” (controversies).
- Much about modal logic and determinism.
- Philosophy had three parts: physics, ethics, and logic.
- Eventually merged with the “traditional logic” based on Aristotle.

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The Medievals developed logic further.

- Boethius: modal logic and translations of Aristotle.
- Arab logic and rediscovery of Aristotle's writings.
- Refined the details: Barbara-Celarent, rules for valid syllogisms, university textbooks.
- Modal logic, problem of universals, etc.
- Much use of logic in philosophy (e.g. Thomas Aquinas); we still use many Latin terms in logic (like "modus ponens" and "a priori").

Renaissance to 19th century logic

- The Enlightenment (e.g. Kant) brought little progress in logic.
- Leibniz proposed a symbolic logic that would reduce reasoning to a kind of calculation.
- Hegel and Marx proposed a *dialectical logic*.
- Others (like De Morgan) proposed new ways to symbolize logical operations.

George Boole (1815–64) invented "mathematical logic."

- Letters stand for sets. So "H" might stand for the set of humans" – and "HM" might stand for the set of entities that are both humans and mortals. Then "all humans are mortals" is "H = HM."
- We can symbolize a syllogism as a series of equations and validate it algebraically:

All humans are mortal.	H = HM
All Greeks are humans.	G = GH
∴ All Greeks are mortal.	∴ G = GM

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- "Boolean algebra" can be interpreted to be about sets or about statements. For example:

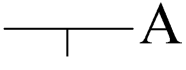
-A	The set of non-As	Not-A
$A \cap B$	The intersection of sets A and B	A and B
$A \cup B$	The union of sets A and B	A or B

- Similar laws cover both interpretations, like "$A \cap B = B \cap A$" and "$-(A \cap B) = (-A \cup -B)$."
- Boole wanted mathematicians to take over logic. But logic today is studied in philosophy, mathematics, and computer science departments.

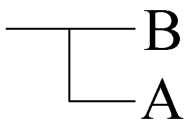
Gottlob Frege (1848–1925) invented "classical symbolic logic."

- This overcame the gap between Aristotelian and Stoic logic in a higher synthesis.
- Frege used lines for "not," "if-then," and "all":

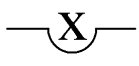
Not-A



If A then B



For all x



Not all A is non-B
(Some A is B)

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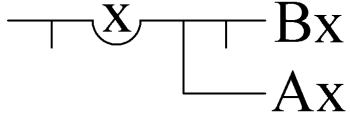

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- Frege tried to show that arithmetic was reducible to logic: every arithmetic truth can be formulated using just notions of logic and proved using just axioms and inference rules of logic.
- One of Frege's axioms said that every condition on x (like "x is a cat") picks out a set. But then "x is not a member of itself" ($x \notin x$) picks out a set R containing just those things that are not members of themselves. Then:

For all x, $x \in R$ if and only if $x \notin x$.
Therefore, $R \in R$ if and only if $R \notin R$.

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Bertrand Russell (1872–1970) tried to put classical logic on a firmer basis.

- *Principia Mathematica* (1910–1913, co-authored with Alfred North Whitehead) used a better notation and tried to avoid "Russell's Paradox."
- This became the standard formulation of *classical symbolic logic* – which gradually replaced *traditional logic* to became the new orthodoxy.

After Principia

- Wittgenstein created truth tables, which gave a *semantic* test of validity.
- Principia's systematization of propositional and quantificational logic was later shown to be sound and complete.
- Kurt Gödel in 1931 showed, against Frege and Russell, that arithmetic was not reducible to any sound and complete axiomatic system.
- Classical logic played a big role in the birth of modern computers.

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Non-classical logics arose.

- *Supplementary non-classical logics* include modal, deontic, belief, and temporal logics.
- *Deviant non-classical logics* include multi-valued and paraconsistent logic.

Logic “in a broad sense” is much pursued.

- *Informal logic* covers various non-formal skills that we need to appraise reasoning.
- *Inductive logic* is about forms of reasoning in which we extrapolate from observed patterns to conclude that a given conclusion is probably true.
- *Metalogic* is the study of formal systems.
- *Philosophy of logic* deals with wider philosophical issues raised by logic (like “What is truth?” and “Are there abstract entities?”).