

Galactic Travel

System S5 lets us go from “ $\square A$ ” in any world W1 to “A” in any world W2.

Weaker systems require a suitable travel ticket between the worlds.
We get travel tickets when you drop diamonds – and we use them when dropping boxes.

System T

We need a *ticket* from W1 to W2.

System S4

Like T, but we also can use a *series* of tickets.

System B

Like T, but a ticket also works *backwards*.

- 1 $\Box(N \supset \Box N)$
- * 2 $\Diamond N$
- [$\therefore \Box N$
- * 3 $\text{asm: } \sim \Box N$
- * 4 $\therefore \Diamond \sim N$ {from 3}
- * 5 $W \therefore N$ {from 2} # \Rightarrow W
- 6 $WW \therefore \sim N$ {from 4} # \Rightarrow WW
- * 7 $W \therefore (N \supset \Box N)$ {from 1} T or any other system
- 8 $W \therefore \Box N$ {from 5 and 7}
- 9 $WW \therefore N$ {from 8} Need S5
- 10 $\therefore \Box N$ {from 3; 6 contradicts 9}

In dropping “ \Box ” (steps 8 to 9), we need to use a *series* of tickets (and one *backwards*), from W to WW.

1	$\Box A$				
	$[\therefore \Box \Box A$				
*	2	asm: $\sim \Box \Box A$			
*	3	$\therefore \Diamond \sim \Box A$ {from 2}			
*	4	$W \therefore \sim \Box A$ {from 3}		$\# \Rightarrow W$	
*	5	$W \therefore \Diamond \sim A$ {from 4}			
	6	$WW \therefore \sim A$ {from 5}		$W \Rightarrow WW$	
	7	$WW \therefore A$ {from 1}		Need S4 or S5	
	8	$\therefore \Box \Box A$ {from 2; 6 contradicts 7}			

In dropping “ \Box ” (from 1 to 7), we need to use a *series* of tickets, from # to WW.

1	A		
	[$\therefore \Box \Diamond A$		
*	2	asm: $\sim \Box \Diamond A$	
*	3	$\therefore \Diamond \sim \Diamond A$ {from 2}	
*	4	W $\therefore \sim \Diamond A$ {from 3}	$\# \Rightarrow W$
	5	W $\therefore \Box \sim A$ {from 4}	
	6	$\therefore \sim A$ {from 5}	Need B or S5
	7	$\therefore \Box \Diamond A$ {from 2; 1 contradicts 6}	

In dropping “ \Box ” (from 5 to 6), we
need to use a ticket *backwards*.

- * 1 $\diamond A$
 $[\therefore \Box \diamond A$
- * 2 $\text{asm: } \sim \Box \diamond A$
- * 3 $\therefore \diamond \sim \diamond A \quad \{\text{from 2}\}$
- * 4 $W \therefore \sim \diamond A \quad \{\text{from 3}\} \quad \# \Rightarrow W$
- 5 $W \therefore \Box \sim A \quad \{\text{from 4}\}$
- 6 $WW \therefore A \quad \{\text{from 1}\} \quad \# \Rightarrow WW$
- 7 $WW \therefore \sim A \quad \{\text{from 5}\} \quad \text{Need S5}$
- 8 $\therefore \Box \diamond A \quad \{\text{from 2; 6 contradicts 7}\}$

In dropping “ \Box ” (steps 5 to 7), we need to use a *series* of tickets (and one *backwards*), from W to WW.

Quantified Modal Translations

$\diamond(\exists x)Ax$ = It's possible for *someone* to be above average.

$\diamond(x)Ax$ = It's possible for *everyone* to be above average.

$(x)\diamond Ax$ = It's possible for *anyone* to be above average.

$\square Fx$ = F is a necessary (essential) property of x.
= x is necessarily F.

$(Fx \cdot \diamond \sim Fx)$ = F is a contingent (accidental) property of x.
= x is F but could have lacked F.

This ambiguous sentence could
have either of two meanings:

“All bachelors are necessarily unmarried.”

Simple Necessity

$$(x)(Bx \supset \Box Ux)$$

All bachelors are *inherently*
unmarriageable – in no possible
world would anyone ever marry
them. (We hope this is false.)

Conditional Necessity

$$\Box(x)(Bx \supset Ux)$$

It’s necessarily true that all
bachelors are unmarried. (This
is trivially true because “bachelor”
means *unmarried man*.)

- $\diamond(\exists x)Ax$ = It's possible for *someone* to be above average.
- $\diamond(x)Ax$ = It's possible for *everyone* to be above average.
- $(x)\diamond Ax$ = It's possible for *anyone* to be above average.

$\square Fx$ = F is a necessary (essential) property of x.
 = x is necessarily F.

$(Fx \cdot \diamond \sim Fx)$ = F is a contingent (accidental) property of x.
 = x is F but could have lacked F.

All bachelors are necessarily unmarried = $(x)(Bx \supset \square Ux)$
 or
 $\square(x)(Bx \supset Ux)$

1 $\Box(x)x=x$ Valid
 [$\therefore (x)\Box x=x$
 * 2 $\text{asm: } \sim(x)\Box x=x$
 * 3 $\therefore (\exists x)\sim\Box x=x$ {from 2}
 * 4 $\therefore \sim\Box a=a$ {from 3}
 * 5 $\therefore \Diamond \sim a=a$ {from 4}
 6 $W \therefore \sim a=a$ {from 5}
 7 $W \therefore (x)x=x$ {from 1}
 8 $W \therefore a=a$ {from 7}
 9 $\therefore (x)\Box x=x$ {from 2; 6 contradicts 8}

1. Reverse squiggles (modal and quantificational).
2. Drop weak operators (modal and quantificational).
3. Lastly, drop strong operators (modal and quantificational).

All bachelors are necessarily unmarried.
 You're a bachelor.
 \therefore "You're unmarried" is logically necessary.

1 $(x)(Bx \supset \Box Ux)$ Valid
 2 Bu
 [$\therefore \Box Uu$
 3 asm: $\sim \Box Uu$
 * 4 $\therefore (Bu \supset \Box Uu)$ {from 1}
 5 $\therefore \Box Uu$ {from 4 and 2}
 6 $\therefore \Box Uu$ {from 3; 3 contradicts 5}

(While this is valid,
 premise 1 is false.)

1 $\Box(x)(Bx \supset Ux)$ Invalid
 2 Bu
 [$\therefore \Box Uu$
 * 3 asm: $\sim \Box Uu$ W
 * 4 $\therefore \Diamond \sim Uu$ {from 3}
 5 W $\therefore \sim Uu$ {from 4}
 6 W $\therefore (x)(Bx \supset Ux)$ {from 1}
 7 $\therefore (x)(Bx \supset Ux)$ {from 1}
 * 8 W $\therefore (Bu \supset Uu)$ {from 6}
 * 9 $\therefore (Bu \supset Uu)$ {from 7}
 10 W $\therefore \sim Bu$ {from 5 and 8}
 11 $\therefore Uu$ {from 2 and 9}

Bu, Uu
$\sim Bu, \sim Uu$

It's possible for anyone to be above average. $(x)\diamond Ax$
 \therefore It's possible for everyone to be above average. $\therefore \diamond(x)Ax$

This lead into an endless loop.
 Using ingenuity, we can devise a refutation:

	a, b
W	Aa, ~Ab
WW	Ab, ~Aa

Problem: Our system doesn't recognize the ambiguity here:

The number I'm thinking of is necessarily odd = $\Box O_n$

I'm thinking of just one number, and it has the necessary property of being odd. (Perhaps true!)

$$(\exists x)((Tx \cdot \sim(\exists y)(\sim x=y \cdot Ty)) \cdot \Box O_x)$$

This is necessary: "I'm thinking of just one number and it is odd." (False!)

$$\Box(\exists x)((Tx \cdot \sim(\exists y)(\sim x=y \cdot Ty)) \cdot O_x)$$

Solution: Analyze "the ..." using Russell's theory of descriptions.

Problem: Our system makes this valid – which it isn't!

$e=n$	8 is the number I'm thinking of.
$\square e=e$	It's necessary that 8 is 8.
$\therefore \square e=n$	\therefore It's necessary that 8 is the number I'm thinking of.

Solution: Analyze “the ...”
using Russell's theory of descriptions.

Problem: Our system makes every entity a necessary being! Here's a proof that in every possible world there exists a being who is Gensler:

	$\therefore \Box(\exists x)x=g$		Valid ???
*	1	asm:	$\sim\Box(\exists x)x=g$
*	2	$\therefore \Diamond\sim(\exists x)x=g$	{ from 1 }
*	3	W $\therefore \sim(\exists x)x=g$	{ from 2 }
	4	W $\therefore (x)\sim x=g$	{ from 3 }
	5	W $\therefore \sim g=g$	{ from 4 } ← ???
	6	W $\therefore g=g$	{ self-identity rule }
	7	$\therefore \Box(\exists x)x=g$	{ from 1; 5 contradicts 6 }

Solution: Reject the step from 4 to 5. Move to a “free logic” – one that is free of the assumption that individual constants like “g” always refer to existing beings.

Free logic reformulates the rules
for dropping “ $(\exists x)$ ” and “ (x) ”:

$(\exists x)Fx \rightarrow Fa, (\exists x)x=a,$
use a *new* constant

Some existing being is F.
 \therefore a is F.
 \therefore a is an existing being.

$(x)Fx, (\exists x)x=a \rightarrow Fa,$
use *any* constant

Every existing being is F.
a is an existing being.
 \therefore a is F.

Our sophisticated system uses “free logic” rules (free of the assumption that constants like “g” refer to existing beings) for dropping “(x)” & “(∃x)”:

$$\boxed{(\forall x)Fx, (\exists x)x=a \rightarrow Fa, \text{ use any constant}}$$

Every existing being is F.
a is an existing being.
∴ a is F.

$$\boxed{(\exists x)Fx \rightarrow Fa, (\exists x)x=a, \text{ use a new constant}}$$

Some existing being is F.
∴ a is F.
∴ a is an existing being.

- [∴ □(∃x)x=g Valid ???
- * 1 asm: ~□(∃x)x=g
 - * 2 ∴ ◇~(∃x)x=g {from 1}
 - * 3 W ∴ ~(∃x)x=g {from 2}
 - 4 W ∴ (x)~x=g {from 3}
 - 5 W ∴ ~g=g {from 4} ← ???
 - 6 W ∴ g=g {self-identity rule}
 - 7 ∴ □(∃x)x=g {from 1; 5 contradicts 6}

Without free logic, every entity (e.g. Gensler) is a necessary being that exists in every possible world. Free logic rejects step 5 in the proof to the left. Now what entities exist can vary from world to world.

Our sophisticated system always uses Russell's theory of descriptions to analyze definite descriptions (terms of the form "the so and so"). This invalidates the following invalid argument:

$e=n$	8 is the number I'm thinking of.
$\Box e=e$	It's necessary that 8 is 8.
$\therefore \Box e=n$	\therefore It's necessary that 8 is the number I'm thinking of.

This also enables us to express the two senses of "The number I'm thinking of is necessarily odd" (which in naïve QMC is just $\Box On$):

- This is necessary: "I'm thinking of just one number and it is odd."
(False!) $\Box(\exists x)((Tx \cdot \sim(\exists y)(\sim x=y \cdot Ty)) \cdot Ox)$
- I'm thinking of just one number, and it has the necessary property of being odd. (Perhaps true!) $(\exists x)((Tx \cdot \sim(\exists y)(\sim x=y \cdot Ty)) \cdot \Box Ox)$