

Quantificational Logic

I_r = Romeo is Italian.

I_x = x is Italian.

$(\forall x)I_x$ = For all x , x is Italian.
= All are Italian.

$(\exists x)I_x$ = For some x , x is Italian.
= Some are Italian.

Use capital letters for *general terms* (terms that *describe* or put in a *category*):

B = a cute baby

C = charming

R = rides a bicycle



Use small letters for *singular terms* (terms that pick out a *specific* person or thing):

b = the world's cutest baby

c = this child

w = William Gensler



A capital letter alone (not followed by small letters) represents a *statement*.

S = *It is snowing.*

A capital letter followed by a single small letter represents a *general term*.

Ir = Romeo is *Italian.*

A small letter from “a” to “w”
is a *constant* – and stands for a
specific person or thing.

$Ir = \textit{Romeo}$ is Italian.

A small letter from “x” to “z” is
a *variable* – and doesn’t stand
for a specific person or thing.

$Ix = x$ is Italian.

“(x)” is a *universal quantifier*. It claims that the formula that follows is true for *all* values of x.

$(x)Ix$ = For all x, x is Italian.
= All are Italian.

“($\exists x$)” is an *existential quantifier*. It claims that the formula that follows is true for *at least one* value of x.

$(\exists x)Ix$ = For some x, x is Italian.
= Some are Italian.

1. The result of writing a capital letter and then a small letter is a wff.
2. The result of writing a quantifier and then a wff is a wff.

If the English begins with

all (every)
not all (not every)
some
no

then begin the wff with

(x)
$\sim(x)$
$(\exists x)$
$\sim(\exists x)$

All are Italian = $(x)Ix$

Not all are Italian = $\sim(x)Ix$

Some are Italian = $(\exists x)Ix$

No one is Italian = $\sim(\exists x)Ix$

All are rich or Italian = $(x)(Rx \vee Ix)$

Not everyone is non-Italian = $\sim(x)\sim Ix$

Some aren't rich = $(\exists x)\sim Rx$

No one is rich and non-Italian = $\sim(\exists x)(Rx \cdot \sim Ix)$

If the sentence doesn't specify the connective:

with “all ... is ...,” use “ \supset ”
for the *middle* connective.

otherwise use “ \cdot ”
for the connective.

All Italians are lovers = $(x)(Ix \supset Lx)$
= For all x, *if* x is Italian *then* x is a lover.

Some Italians are lovers = $(\exists x)(Ix \cdot Lx)$
= For some x, x is Italian *and* x is a lover.

No Italians are lovers = $\sim(\exists x)(Ix \cdot Lx)$
= It is not the case that, for some x, x is
Italian *and* x is a lover.

All rich Italians are lovers = $(x)((Rx \cdot Ix) \supset Lx)$
= For all x, *if* x is rich *and* Italian, *then* x
is a lover.

Quantificational Logic

I_r = Romeo is Italian.

I_x = x is Italian.

$(\forall x)I_x$ = All are Italian = For all x , x is Italian.

$(\exists x)I_x$ = Some are Italian = For some x , x is Italian.

If the English begins with:	→	all (every)	not all	some	no
then begin the wff with:	→	$(\forall x)$	$\sim(\forall x)$	$(\exists x)$	$\sim(\exists x)$

With “all ... is ...,” use “ \supset ”
for the *middle* connective.

Otherwise use “ \cdot ”
for the connective.

Quantificational Inference Rules

First reverse
squiggles

$$\begin{array}{l} \sim(x)Fx \rightarrow (\exists x)\sim Fx \\ \sim(\exists x)Fx \rightarrow (x)\sim Fx \end{array}$$

*

and drop
existentials.

$$(\exists x)Fx \rightarrow Fa,$$

use a *new* constant

*

Lastly, drop
universals.

$$(x)Fx \rightarrow Fa,$$

use any constant

Don't
star

Valid

- 1 $(x)(Fx \cdot Gx)$
[$\therefore (x)Fx$
- * 2 asm: $\sim(x)Fx$
- * 3 $\therefore (\exists x)\sim Fx$ {from 2} ← reverse squiggles
- 4 $\therefore \sim Fa$ {from 3} ← drop existentials
- 5 $\therefore (Fa \cdot Ga)$ {from 1} ← drop universals
- 6 $\therefore Fa$ {from 5}
- 7 $\therefore (x)Fx$ {from 2; 4 contradicts 6}

1. Reverse squiggles.
2. Drop initial existentials, using a new letter each time.
3. Lastly, drop each initial universal once for each old letter. (Only use a new letter if you've done everything else possible, including further assumptions if needed, and still have no old letters.)

- 1 $(x)(Lx \supset Fx)$
- * 2 $(\exists x)Lx$
[$\therefore (x)Fx$
- * 3 asm: $\sim(x)Fx$
- * 4 $\therefore (\exists x)\sim Fx$ {from 3}
- 5 $\therefore La$ {from 2}
- 6 $\therefore \sim Fb$ {from 4}
- * 7 $\therefore (La \supset Fa)$ {from 1}
- * 8 $\therefore (Lb \supset Fb)$ {from 1}
- 9 $\therefore Fa$ {from 5 and 7}
- 10 $\therefore \sim Lb$ {from 6 and 8}

Invalid

a, b

La, Fa
 $\sim Lb, \sim Fb$

Reverse squiggles, drop existentials, drop universals.
 If you can't get a contradiction, construct a refutation.

$$1 \quad (x)(Lx \supset Fx) = 1$$

$$2 \quad (\exists x)Lx = 1$$

$$[\therefore (x)Fx = 0$$

Invalid

a, b

La, Fa
 \sim Lb, \sim Fb

An *existential* wff is true if and only if *at least one case* is true.

A *universal* wff is true if and only if *all cases* are true.

If a wff doesn't start with a quantifier: evaluate each subformula that starts with a quantifier, and then substitute "1" or "0" for it:

$$\begin{aligned} & \sim(x)Fx \\ = & \sim 0 \\ = & 1 \end{aligned}$$

$$\begin{aligned} & \sim(x)(Lx \supset Fx) \\ = & \sim 1 \\ = & 0 \end{aligned}$$

$$\begin{aligned} & ((\exists x)Fx \supset (x)Lx) \\ = & (1 \supset 0) \\ = & 0 \end{aligned}$$

1 (x)(Fx · Gx) Valid
 [∴ (x)Fx
 * 2 asm: ∼(x)Fx
 * 3 ∴ (∃x)∼Fx {from 2}
 4 ∴ ∼Fa {from 3}
 5 ∴ (Fa · Ga) {from 1}
 6 ∴ Fa {from 5}
 7 ∴ (x)Fx {from 2; 4 contradicts 6}

1 (x)(Lx ⊃ Fx) Invalid
 a, b
 * 2 (∃x)Lx
 [∴ (x)Fx
 * 3 asm: ∼(x)Fx
 * 4 ∴ (∃x)∼Fx {from 3} La, Fa
∼Lb, ∼Fb
 5 ∴ La {from 2}
 6 ∴ ∼Fb {from 4}
 * 7 ∴ (La ⊃ Fa) {from 1}
 * 8 ∴ (Lb ⊃ Fb) {from 1}
 9 ∴ Fa {from 5 and 7}
 10 ∴ ∼Lb {from 6 and 8}

1. everse squiggles.
2. Drop initial existentials, using a new letter each time.
3. Lastly, drop each initial universal once for each old letter. (Only use a new letter if you've done everything else possible, including further assumptions if needed, and still have no old letters.)
4. If you can't get a contradiction, construct a refutation.

Harder translations add statement letters, individual constants, and non-initial or multiple quantifiers:

$(S \supset Cr)$ = If it's snowing, then Romeo is cold.

$((x)Ix \supset (x)Lx)$ = If all are Italian, then all are lovers.

Use a separate quantifier for each “all,” “some,” and “no”; and place the quantifiers to mirror where they occur in English:

Wherever the English has

put this in the wff

all (every)
not all (not every)
some
no

(x)
$\sim(x)$
$(\exists x)$
$\sim(\exists x)$

Translate this right now:

“If all Greeks are mortal and
Socrates is Greek, then someone
is mortal and it will rain.”

$((\forall x)(Gx \supset Mx) \cdot Gs) \supset ((\exists x)Mx \cdot R)$

“If all Greeks are mortal and Socrates is Greek, then someone is mortal and it will rain.”

To translate “any”:

First rephrase the sentence so it means the same thing but doesn't use “any”; then translate the second sentence.

or

Put a “(x)” at the *beginning* of the wff, regardless of where “any” occurs in the sentence.

Not anyone is rich = $\sim(\exists x)Rx$ = $(x)\sim Rx$

Not any Italian is a lover = $\sim(\exists x)(Ix \cdot Lx)$ = $(x)\sim(Ix \cdot Lx)$

If anyone is just, there will be peace = $((\exists x)Jx \supset P)$ = $(x)(Jx \supset P)$

Harder translations add statement letters, individual constants, and non-initial or multiple quantifiers:

$$(S \supset Cr)$$

$$((x)Ix \supset (x)Lx)$$

Wherever the English has:	→	all (every)	not all	some	no
put this in the formula:	→	(x)	~(x)	(∃x)	~(∃x)

With “all ... is ...,” use “ \supset ” for the *middle* connective.

Otherwise use “ \cdot ” for the connective.

To translate a sentence with “any”:

- First rephrase the sentence so it means the same thing but doesn’t use “any”; then translate the second sentence
- OR: Put a “(x)” at the *beginning* of the wff, regardless of where “any” occurs in the sentence.

Proofs with harder formulas:

- use statement letters, individual constants, or non-initial or multiple quantifiers,
- often require multiple assumptions, but
- require no new inference rules.

Remember to
drop only initial
quantifiers.

“ $((x)Fx \supset (x)Gx)$ ”
is an if-then and follows
the if-then rules.

The Two Great Commandments:

- (1) Thou shalt drop only initial quantifiers.
- (2) Thou shalt use a new letter when dropping ($\exists x$).

Do this problem now:

$$\sim(x)(Fx \vee Sx)$$
$$\therefore ((\exists x)\sim Sx \cdot \sim Fa)$$

1. Reverse squiggles.
2. Drop initial existentials, using a new letter each time.
3. Lastly, drop each initial universal once for each old letter. (Only use a new letter if you've done everything else possible, including further assumptions if needed, and still have no old letters.)
4. If you can't get a contradiction, construct a refutation.